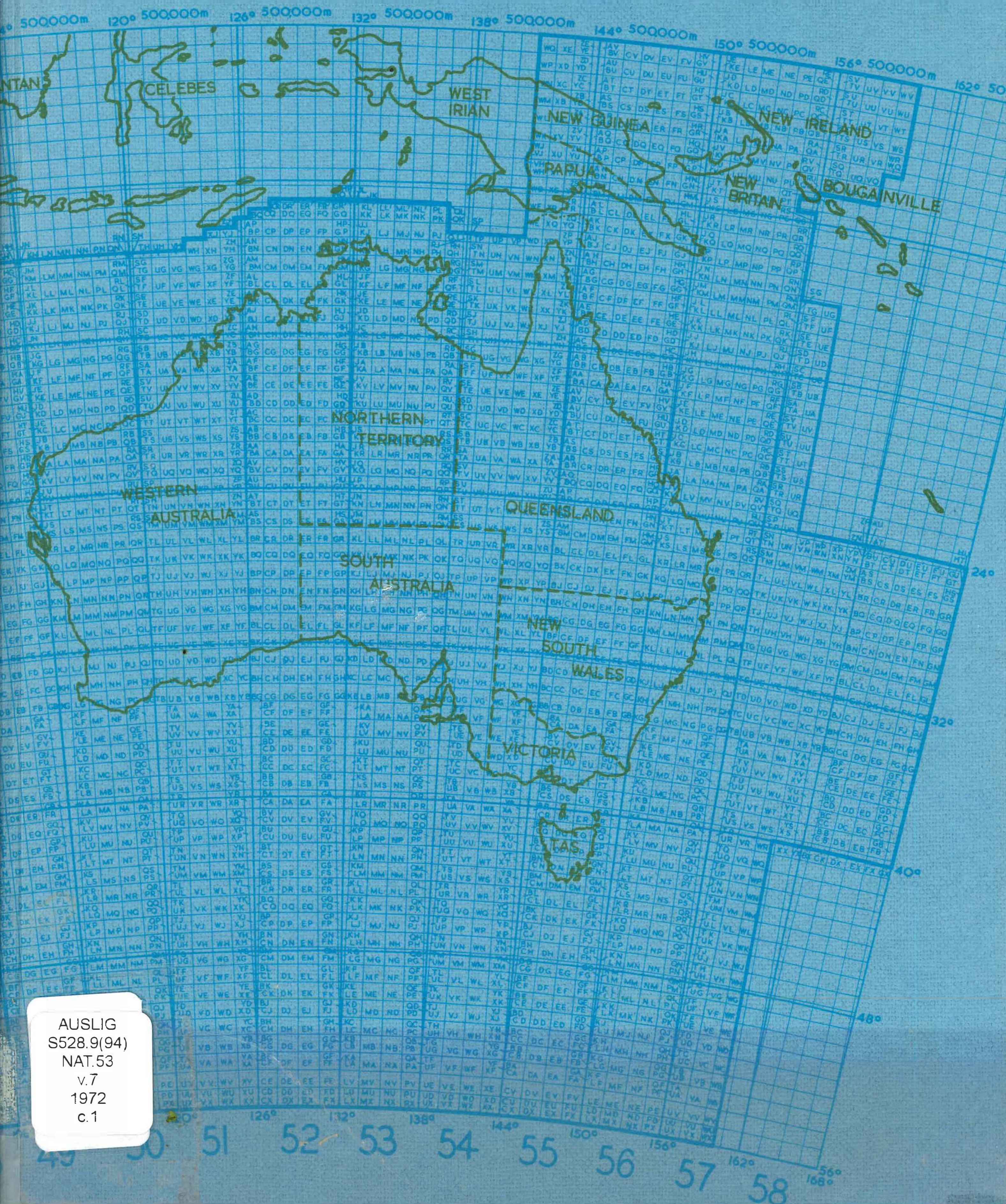
The Australian Map Grid

TECHNICAL MANUAL

NATIONAL MAPPING COUNCIL OF AUSTRALIA

Special Publication 7



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The Australian Map Grid

TECHNICAL MANUAL

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Foreword

At its 24th meeting in Melbourne in April 1966, the National Mapping Council, in respect of the Australian Map Grid, agreed that a working party should be set up to consider and recommend suitable definitions, sign conventions and symbols; that the party should consist of a member from Victoria (Convener); a member from the Division of National Mapping; a member of the Royal Australian Survey Corps; and a member from New South Wales; and that the working party should be empowered to seek the assistance of a representative from the University of Melbourne.

The Council also decided that a detailed booklet setting out full technical details of the new grid, including definitions and formulae, be prepared by the Director of National Mapping.

The members of the Working Party (with their qualifications at that time) were:

J. E. Mitchell, L.S., M.I.S. Aust., M.A.I.C. Convener
Department of Crown Lands and Survey, Melbourne
A. G. Bomford, M.A., A.R.I.C.S., M.I.S. Aust., M.A.I.C.
Division of National Mapping, Canberra
Major N. R. J. Hillier (to January 1967)
Royal Australian Survey Corps
Major H. Taylor, L.S., M.I.S. Aust., A.M.A.I.C. (from January 1967)
Royal Australian Survey Corps
B. Purins, B.Surv., A.M.I.S., Aust., M.A.I.C.
Department of Lands, Sydney
B. T. Murphy, M.Surv., L.S., F.I.S., Aust.
Department of Surveying, University of Melbourne

The working party met at intervals over three and a half years. Their discussions were thorough. Few errors were discovered in the first edition of the Manual and these have been corrected in this second edition.

The National Mapping Council at its 28th meeting, in Hobart 1970, asked me to convey its thanks to the working party, and this I had pleasure in doing on 1 December 1970.

I would like to thank the Australian Government Publishing Service for publishing this second edition of the Manual in a manner which its contents deserve.

B. P. Lambert

(B. P. LAMBERT)
Chairman, National Mapping Council

Canberra
August 1971

Contents

1 General Notes

1.1 INTRODUCTION

1.1.1 In 1966, all the geodetic surveys in Australia and New Guinea were recomputed and adjusted on the new Australian Geodetic Datum (AGD). There is now a homogeneous system of coordinates for geodetic survey stations all over Australia, free from discontinuities caused by changes of origin. Coordinates on the Australian Geodetic Datum provide a firm foundation on which lower order surveys and all mapping can be based; furthermore, they provide a basis for a point reference system, on which any point in Australia – for example, an oil well, a mineral deposit, a civil defence headquarters, or a bush fire outbreak – can be described in precise and unambiguous terms.

1.1.2 Geodetic coordinates are usually computed in latitude and longitude. For many purposes, including mapping, a system of rectangular grid coordinates – eastings and northings – is more convenient. A Transverse Mercator projection has long been used in Australia for this purpose, but the opportunity has been taken to change from the old Australian National Grid in yards, to the Universal Transverse Mercator Grid in metres, which is in world-wide use. The new grid is called the *Australian Map Grid* (AMG). At the same time, all distances and heights, previously quoted in yards or feet, have been converted to metres, in accordance with the policy of a gradual change to the metric system.

1.1.3 The aims of this manual are:

1.1.3.1 To define the Australian Geodetic Datum and the Australian Map Grid;

1.1.3.2 To define standard symbols, terms and sign conventions for use throughout Australia;

1.1.3.3 To provide a set of numerical examples, using the full rigour of the defining formulae, as a standard against which computer programs and other more convenient but less accurate formulae can be judged;

1.1.3.4 To provide standard first-order formulae for routine computation, each with numerical examples;

1.1.3.5 To provide simpler approximate formulae, with numerical examples, for use on surveys of limited extent or of lower accuracy;

1.1.3.6 To show how to use grid references.

1.1.4 Logarithmic methods are not considered in this manual.

1.2 THE AUSTRALIAN GEODETIC DATUM

1.2.1 Surveys in Australia are computed on the *Australian National Spheroid,** for which the defining parameters are:

Major semi-axis, $a = 6\ 378\ 160$ metres
Flattening, $f = 1/298{\cdot}25$

Derived functions for this spheroid are given in paragraph *3.2.*

1.2.2 In 1966, the minor axis of the spheroid was defined to be parallel to the earth's mean axis of rotation at the start of 1962. In 1970, the National Mapping Council decided to adopt the Conventional International Origin, previously known as the mean pole of 1901-05, for the

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* The International Union of Geodesy and Geophysics at its meeting in Lucerne in 1967 adopted the Reference Ellipsoid 1967, which is very similar but not identical to the Australian National Spheroid:

	$1/f$	e^2
Australian National Spheroid	298·250 000 000	0·006 694 541 855
Reference Ellipsoid 1967:	298·247 167 427	0·006 694 605 328 57

The difference is trivial, amounting to only 0·07 metres in the minor axis. But it is not possible to formally adopt the Reference Ellipsoid 1967 without recomputing all AGD coordinates.

direction of the minor axis. The Council decided that no change in the 1966 coordinates was necessary. The plane of zero geodetic longitude is defined to be parallel to the vertical through the Bureau International de l'Heure (BIH) mean observatory near Greenwich, that is to say 149°00′18″855 West of the vertical through the photo zenith tube at Mount Stromlo. The position of the centre of the spheroid is defined by the following coordinates of *Johnston Geodetic Station:*

Geodetic Latitude = 25° 56′ 54″551 5 South;
Geodetic Longitude = 133° 12′ 30″077 1 East;
Spheroidal Height = 571·2 metres.

1.2.3 The size, shape, position and orientation of the spheroid are thus completely defined, and together define the *Australian Geodetic Datum.* The coordinates for Johnston Geodetic Station were derived from astronomical observations at 275 stations on the geodetic survey distributed all over Australia. The defining spheroidal height of 571·2 metres is equal to the height of the station above the geoid, as computed by trigonometrical levelling in 1965.

1.2.4 In future years, it may well be that the geodetic surveys in Australia will be readjusted, so that scientists and others interested in the most accurate possible coordinates and distances in Australia may take advantage of all the most recent observations. For mapping, and for references on the Australian Map Grid, it is intended to retain coordinates in terms of the 1966 adjustment. If it becomes necessary to avoid ambiguity, other coordinates should be qualified by a statement of their source.

1.3 THE AUSTRALIAN MAP GRID

1.3.1 Coordinates on the Australian Map Grid are derived from a Transverse Mercator projection of latitudes and longitudes on the Australian Geodetic Datum. The coordinates are defined by the formulae for easting and northing given in paragraph *4.3,* first published by J.C.B. Redfearn in the *Empire Survey Review,* No. 69, 1948. They are correct to less than 1mm anywhere in a grid zone. For the purposes of this definition, these formulae, and the formula for meridian distance given in paragraph *4.2,* are to be regarded as exact, not as the opening terms of infinite series.

1.3.2 The Australian Map Grid corresponds with the Universal Transverse Mercator Grid, as follows:

1.3.2.1 Coordinates are in metres;

1.3.2.2 Zones are 6° wide plus ½° overlaps;

1.3.2.3 AMG zones are numbered from zone 47 with central meridian 99°E to zone 58 with central meridian 165°E;

1.3.2.4 The origin of each zone is the intersection of the central meridian with the equator;

1.3.2.5 A central scale factor, k_0, of 0·999 6 is superimposed on all projected distances;

1.3.2.6 Eastings E are defined by adding 500 000 metres to the value of E′, given by the formula in paragraph *4.3.1;*

1.3.2.7 In the southern hemisphere, northings N are defined by adding 10 000 000 metres to the negative value of N′ given by the formula in paragraph *4.3.2.*

1.4 THE LIMITS OF THE AUSTRALIAN MAP GRID

Although the Universal Transverse Mercator system is of world-wide application, different countries use different spheroids, and it is therefore necessary to define the limits within which the Australian National Spheroid and the Australian Map Grid are to be used. The Australian Map Grid covers Australia and the Territories administered by Australia within the area delineated in Annex A. The grid does not cover Heard Island and the McDonald Islands, nor the Australian Antarctic Territory, for which the International Spheroid remains in use.

1.5 CONVERSION OF FEET AND METRES

Although coordinates on the Australian Map Grid are expressed in metres, some measurements in feet are likely to be made for many years. The conversion ratios laid down in the Weights and

Measures (National Standards) Regulations, 1961, for use in Australia are:

1 Yard = 0·9144 metres exactly
1 Foot = 1/3 yard
whence 1 Foot = 0·3048 metres exactly.

Until 1960 or so, many authorities used 'British feet'. The physical length of the British standard yard has changed significantly over the years. Great care is necessary to determine the correct factor to convert old work from feet to metres.

1.6 THE NUMERICAL EXAMPLES

The various formulae have been tested over a line in Victoria from Flinders Peak to Buninyong. The line is nearly 55 kilometres long in an azimuth of 307°, and runs across the boundary of zones 54 and 55. It thus provides a strong, though not extreme, test of all the formulae. The numerical computations are given in full. A summary of geodetic values for the test line computed from Robbins's and Redfearn's formulae is given in Annex B. The computations and summary should prove useful for testing computer programs, and by comparing the results obtained in Chapters 5 and 6 with the rigorous results from Chapters 3 and 4, the accuracy of the approximate formulae can be checked.

1.7 THE UNITED STATES ARMY TABLES

1.7.1 The United States Army has produced four volumes of tables which assist computations on the Australian Geodetic Datum and transformations to the Australian Map Grid:

TM5-241-32/1 Transformation of Coordinates from Geographic to Grid
TM5-241-32/2 Transformation of Coordinates from Grid to Geographic
TM5-241-33 Latitude Functions
TM5-241-34 Grid Coordinates for 5′ Intersections.

The methods of computation described in Chapters 5 and 6 make use of these tables. The definitions of the tabulated terms (I) to (XVIII) in the two volumes TM5-241-32/1 and /2 are given in Annex C to this manual. The definitions of the terms A, B, C, D, E, F, in TM5-241-33 are given on page ii of the tables.

1.7.2 Similar tables have been produced by the United States Army for the following spheroids:

International Clarke 1866 Bessel
Everest Clarke 1880

1.7.3 The computation forms given in Chapter 5, although designed primarily for use in Australia with the tables for the Australian National Spheroid, can be used with any spheroid for which tables are available.

1.8 COMPUTER PROGRAMS

1.8.1 Many of the computations described in this manual have been programmed for electronic computers by various authorities. The rigorous formulae given in Chapter 3 and Chapter 4 are too laborious to use in any other way.

1.8.2 Surveyors wishing to use programs or obtain copies of data sheets, Fortran listings or other information should contact their State Surveyor General. Details of new programs are published annually in *Electronic Survey Computing* by the National Mapping Council. Copies of this publication are available from the Director of National Mapping.

1.8.3 The rigorous formulae of Chapters 3 and 4 have been programmed by the Division of National Mapping as follows:

CLARKROB – Latitude and Longitude from Distance and Azimuth using Robbins's Formulae – see paragraph *3.3.*

ROBBINS – Distance and Azimuth from Latitude and Longitude using Robbins's Formulae – see paragraph *3.5.*

TMCOORD – Transformation of Geographic Coordinates to Grid or Grid Coordinates to Geographic, using Redfearn's Formulae – see paragraphs *4.3* and *4.5.*

In addition the following programs are available:

LAUF	– Conformal Transformation from Grid to Grid by Lauf's Formulae.
VARYCORD	– A least squares adjustment of horizontal control surveys which gives both AGD and AMG values.

2 Symbols, Definitions and Sign Conventions

2.1 SYMBOLS

The sybmols used with the Australian Map Grid are listed below. Many terms are more fully defined in paragraph *2.2* below, and illustrated in Annex D. Some symbols used with the Australian National Spheroid are listed in paragraph *3.2*.

ϕ = Geodetic latitude, negative south of the equator.

ϕ_1, ϕ_2 = Latitude at points 1 and 2 respectively

ϕ_m = $(\phi_1 + \phi_2) / 2$

$\Delta\phi$ = $\phi_2 - \phi_1$

$\Delta\phi''$ = $\Delta\phi$ expressed in seconds of arc

and similarly for other terms.

λ = Geodetic longitude measured from Greenwich, positive eastwards.

$\Delta\lambda$ = $\lambda_2 - \lambda_1$

λ_0 = Geodetic longitude of a central meridian.

ω = Geodetic longitude measured from a Central Meridian, positive eastwards: $\omega = \lambda - \lambda_0$

E' = Easting measured from a central meridian, positive eastwards.

N' = Northing measured from the equator, negative southwards.

E = $E' + 500\,000$ metres.

N = N' in the northern hemisphere.

= $N' + 10\,000\,000$ metres in the southern hemisphere.

ρ, ν = Radii of curvature of the spheroid in meridian and prime vertical respectively.

α = Azimuth, clockwise through 360° from true north.

β = Grid bearing, clockwise through 360° from grid north.

θ = Plane bearing, clockwise through 360° from grid north.

γ = Grid convergence, positive when grid north is west of true north, negative when grid north is east of true north. $\beta = \alpha + \gamma$

δ = Arc-to-chord correction, with sign defined by the equations: $\theta = \beta + \delta = \alpha + \gamma + \delta$

$\Delta\alpha$ = Meridian convergence.

$\Delta\beta$ = Line curvature.

s = Spheroidal distance.

S = Grid distance.

L = Plane distance.

m = Meridian distance, true distance from the equator, negative southwards.

a, b = Major and minor semi-axes of the spheroid.

e^2 = $(a^2 - b^2) / a^2 = (\text{eccentricity})^2$

e'^2 = $(a^2 - b^2) / b^2 = (\text{second eccentricity})^2$

k_0 = Central scale factor = 0·999 6

k = Point scale factor.

K = Line scale factor.

t = $\tan\phi$

ψ = ν / ρ

ϕ' = Latitude for which $m = N'/k_0$

t', ψ', ρ', ν' are functions of the latitude ϕ'

R^2 = $\rho\nu$

r^2 = $R^2 k_0^2 = \rho\nu k_0^2$

Note: E', N', E, N, S, L, r, k and K include the central scale factor, k_0.

s, ρ, ν, R and m are true distances, which must be specifically multiplied by k_0 when necessary.

2.2 **DEFINITIONS** – see also Annex D.

2.2.1 Azimuth, Meridian Convergence, and Spheroidal Distance

2.2.1.1 *Azimuth,* α, is a horizontal angle measured from the spheroidal meridian clockwise from North through 360°

2.2.1.2 *Meridian Convergence,* $\Delta\alpha$, is the change in the azimuth of a geodesic between two points on the spheroid:

Reverse Azimuth = Forward Azimuth + Meredian Convergence ± 180°

$$\alpha_2 = \alpha_1 + \Delta\alpha \pm 180^{\circ}$$

2.2.1.3 *Spheroidal Distance,* s, is distance on the spheroid along either a normal section or a geodesic. The difference is usually negligible.

2.2.2 Grid Bearing, Line Curvature and Grid Distance

A line on the spheroid of length s is projected on the grid as an arc.

2.2.2.1 *Grid Bearing,* β, at a point on the arc, is the angle between Grid North and the tangent to the arc at the point. It is measured from Grid North clockwise through 360°

2.2.2.2 *Line Curvature,* $\Delta\beta$, is the change in grid bearing between two points on the arc.

Reverse Grid Bearing = Forward Grid Bearing + Line Curvature ± 180°

$$\beta_2 = \beta_1 + \Delta\beta \pm 180^{\circ}$$

2.2.2.3 *Grid Distance,* S, is the length measured along the arc of the projected line whose spheroidal distance is s.

2.2.3 Plane Bearing and Plane Distance

A straight line can be drawn on the grid between the ends of the arc defined in paragraph *2.2.2.*

2.2.3.1 *Plane Bearing,* θ, is the angle between the Grid North and this straight line.

2.2.3.2 *Plane Distance,* L, is the length of this straight line. The difference in length between the plane distance, L, and the grid distance, S, is nearly always negligible.

2.2.3.3 Using plane bearings and plane distances, the formulae of plane trigonometry hold with complete rigour:

$$\tan\theta = \Delta E/\Delta N;\ \Delta E = L\sin\theta;\ \Delta N = L\cos\theta.$$

2.2.4 Grid Convergence

Grid convergence, γ, is the angular quantity to be added algebraically to an azimuth to obtain a grid bearing:

Grid Bearing = Azimuth + Grid Convergence

$$\beta = \alpha + \gamma$$

The sign of the grid convergence is determined by formulae *4.3.3* and *4.5.3.* In the southern hemisphere, grid convergence is positive for points east of a central meridian, and negative west.

2.2.5 Arc-to-Chord Correction

An arc-to-chord correction, δ, is the angular quantity to be added algebraically to a grid bearing to obtain a plane bearing:

$$\theta = \beta + \delta \doteq \alpha + \gamma + \delta$$

The arc-to-chord corrections differ in amount and sign at the two ends of a line. The sign is determined by the formulae in paragraphs *5.16* and *6.10.* Lines which do not cross the central meridian always bow away from the central meridian. Note that

$$\Delta\beta = \delta_1 - \delta_2$$

2.2.6 Scale Factors

2.2.6.1 *Point Scale Factor,* k, is the ratio of an infinitesimal distance at a point on the grid to the corresponding distance on the spheroid:

$$k = dL/ds = dS/ds$$

It is the distinguishing feature of conformal projections, such as the Transverse Mercator, that this ratio is independent of the azimuth of the infinitesimal distance.

2.2.6.2 *Line Scale Factor,* K. From point to point along a line on the grid, the point scale factor will in general vary. The line scale factor is the ratio of a plane distance, L, on the grid to the corresponding spheroidal distance, s:

$$K = L/s \simeq S/s$$

3 Rigorous Formulae on the Spheroid

3.1 AIMS

3.1.1 This chapter gives formulae and numerical examples for highly accurate computations on the spheroid:

3.1.1.1 *The Direct Problem:* latitude and longitude from spheroidal distance and azimuth.

3.1.1.2 *The Reverse Problem:* spheroidal distance and azimuth from latitude and longitude.

The formulae are those given by Dr A.R. Robbins in *Empire Survey Review* No. 125, 1962. They are accurate to better than 20mm over distances of 1 500 kilometres. The errors can reach 16 metres at 4 500 kilometres, and more than 2 000 metres at 9 000 kilometres.

3.1.2 It is not foreseen that these formulae will ever be used for hand computation: they require ten-figure trigonometrical functions, and it is easier and better to use the programs available for electronic computers. In the numerical examples, all trigonometrical functions and intermediate results are given, which should be adequate for checking similar programs; but they have not been laid out in a form suitable for hand computation, and they may give a false idea of ease and brevity.

3.1.3 Robbins's formulae are for the normal section. For conversion to the geodesic, see paragraph *3.7.*

3.2 FUNCTIONS FOR THE AUSTRALIAN NATIONAL SPHEROID

3.2.1 By definition (see paragraph *1.2.1*):

Major semi-axis, a = 6 378 160 metres
Flattening, f = 1/298·25 exactly

3.2.2 From these figures can be derived:

Flattening, f = 0·003 352 891 869
Minor semi-axis, b = a (1–f) = 6 356 774·719 metres
$e^2 = 2f - f^2 = (a^2 - b^2)/a^2$ = 0·006 694 541 855
e = 0·081 820 178 0
$e'^2 = e^2 + e^4/(1-e^2) = (a^2-b^2)/b^2$ = 0·006 739 660 796
e′ = 0·082 095 437 1

3.2.3 For the computation of radii of curvature:

$c = a/(1-e^2)^{1/2}$ = 6 399 617·225 metres
$V^2 = 1+e'^2 \cos^2\phi = \nu/\rho = \psi$ (in Chapter 4)
$\rho = c/V^3$;
$\nu = c/V$;
$R = (\rho\nu)^{1/2} = c/V^2$

3.2.4 For computing meridian distances on the Australian National Spheroid, the formula given in paragraph *4.2* reduces to:

$$m = 111\ 133{\cdot}348\ 785\ \phi^0 - 16\ 038{\cdot}954\ 6 \sin 2\phi + 16{\cdot}833\ 1 \sin 4\phi - 0{\cdot}021\ 8 \sin 6\phi \text{ metres}$$

where ϕ^0 = latitude in degrees

The values of A′, B′, C′, D′ quoted on page i of TM5-241-33 Latitude Functions include higher order terms. The difference in meridian distance is less than 0·5mm in latitude 45°.

3.2.5 The following 11-figure values are often useful:

sin 1″ = 0·000 004 848 136 811 1
π = 3·141 592 653 6

1 radian = 57·295 779 513 degrees
= 3 437·746 770 8 minutes
= 206 264·806 25 seconds

3.3 THE DIRECT PROBLEM: ROBBINS'S FORMULAE

Robbins's formulae work in the normal section. In these paragraphs,

α_{12} is the azimuth at point 1 to point 2 in the normal section at point 1
α_{21} ” ” ” ” 2 ” 1 ” ” ” ” ” 2
α'_{12} ” ” ” ” 1 ” 2 ” ” ” ” ” 2
α'_{21} ” ” ” ” 2 ” 1 ” ” ” ” ” 1

1. $h = e' \cos\phi_1 \cos\alpha_{12}$
2. $g = e' \sin\phi_1$
3. $\eta = s/\nu_1$
4. $\sigma = \eta\,[\![1 + \eta^2 h^2(1-h^2)/6 - \eta^3 gh\,(1-2h^2)/8 - (n^4/120)\,[h^2(4-17h^2) - 3g^2(1-7h^2)] + \eta^5 gh/48]\!]$

5a. $\sin\zeta_2 = \sin\phi_1 \cos\sigma + \cos\phi_1 \cos\alpha_{12} \sin\sigma$
6a. $\sin\Delta\lambda = \sin\sigma \sin\alpha_{12}/\cos\zeta_2$
7a. $\sin\alpha'21 = -\cos\phi_1 \sin\alpha_{12}/\cos\zeta_2$

If any of $\zeta_2, \Delta\lambda$ or α'_{21} approach 90° then use the following three formulae:

5b. $\cot\Delta\lambda = (\cos\phi_1 \cot\sigma - \sin\phi_1 \cos\alpha_{12})/\sin\alpha_{12}$
6b. $\tan\zeta_2 = (\sin\phi_1 \cos\Delta\lambda + \sin\Delta\lambda \cot\alpha_{12})/\cos\phi_1$
7b. $\cot\alpha'_{21} = (\cos\sigma \cos\alpha_{12} - \sin\sigma \tan\phi_1)/\sin\alpha_{12}$

Then:

8. $\mu = 1 + \tfrac{1}{2}e'^2\,(\sin\zeta_2 - \sin\phi_1)^2$
9. $\tan\phi_2 = \tan\zeta_2\,(1 + e'^2)\,(1 - e^2\,\mu \sin\phi_1/\sin\zeta_2)$
10. $\lambda_2 = \lambda_1 + \Delta\lambda$
11. $\alpha_{21} = \alpha'_{21} - (\phi_2 - \zeta_2)\sin\alpha'_{21}\tan(\sigma/2)$

3.4 ROBBINS'S DIRECT FORMULAE: NUMERICAL EXAMPLE

From Buninyong. ϕ_1 = – 37° 39′ 15″·557 1
λ_1 = + 143° 55′ 30″·633 0
To Flinders Peak s = 54 972·161 metres
α_{12} = 127° 10′ 27″·08

$\sin\phi_1$ = – 0·610 896 049 5
$\cos\phi_1$ = + 0·791 710 816 3
$\sin\alpha_{12}$ = + 0·796 802 202 4
$\cos\alpha_{12}$ = – 0·604 240 225 6

ν_1 = 6 386 142·439
h = – 0·039 273
g = – 0·050 152
η = 0·008 608 038 7

$+\eta^2$ term . = + $1{\cdot}902.10^{-8}$
$-\eta^3$ term = – $1{\cdot}566.10^{-10}$
$-\eta^4$ term = + $6{\cdot}109.10^{-14}$
$+\eta^5$ term = + $1{\cdot}940.10^{-15}$
Sum = + $1{\cdot}886.10^{-8}$

σ radians = 0·008 608 038 8
σ = 00° 29′ 35″·535 5

$\sin\sigma$ = 0·008 607 932 5
$\cos\sigma$ = 0·999 962 951 1

$\sin\zeta_2$ = – 0·614 991 309 5
ζ_2 = – 37° 57′ 04″·639 8
$\cos\zeta_2$ = + 0·788 533 885 9
$\tan\zeta_2$ = – 0·779 917 414 5

$\sin\Delta\lambda$ = + 0·008 698 192 6
$\Delta\lambda$ = + 00° 29′ 54″·153 6
λ_1 = 143° 55′ 30″·633 0
λ_2 = **144° 25′ 24″·786 6**

$\sin\alpha'_{21}$ = – 0·800 012 445 1
α'_{21} = 306° 52′ 07″·35

μ = 1·000 000 056 5

$\tan\phi_2$ = – 0·779 952 416 7
ϕ_2 = **– 37° 57′ 09″·128 8**

$\phi_2 - \zeta_2$ = –04″·489 0
$\sigma/2$ = + 00° 14′ 47″·767 7
$\tan(\sigma/2)$ = 0·004 304
α_{21} = **306° 52′ 07″·34**

3.5 THE REVERSE PROBLEM: ROBBINS'S FORMULAE

1. $\tan \zeta_2 = (1 - e^2) \tan \phi_2 + e^2 \nu_1 \sin \phi_1 / (\nu_2 \cos \phi_2)$
2. $\tau_1 = \cos \phi_1 \; \tan \zeta_2 - \sin \phi_1 \; \cos (\lambda_2 - \lambda_1)$
3. $\tan \alpha_{12} = \sin (\lambda_2 - \lambda_1)/\tau_1$
4. Obtain α_{21} from formulae 1-3 with suffixes 1 and 2 interchanged.
5. $\chi = \sin (\lambda_2 - \lambda_1)/\sin \alpha_{12}$, unless $\sin \alpha_{12}$ is close to zero, in which case prefer:
   $\chi = \tau_1 / \cos \alpha_{12}$
6. $\sin \sigma = \chi \cos \zeta_2$
7. $g = e' \sin \phi_1$
8. $h = e' \cos \phi_1 \cos \alpha_{12}$
9. $s = \nu_1 \sigma \llbracket 1 - \sigma^2 h^2 (1-h^2)/6 + \sigma^3 gh (1-2h^2)/8 + \sigma^4 \; [h^2(4-7h^2) - 3g^2 (1-7h^2)]/120 - \sigma^5 gh/48 \rrbracket$

3.6 ROBBINS'S REVERSE FORMULAE: NUMERICAL EXAMPLE

Station:		*1. Buninyong*	*2. Flinders Peak*
ϕ	=	− 37° 39′ 15″·557 1	− 37° 57′ 09″·128 8
λ	=	+ 143° 55′ 30″·633 0	+ 144° 25′ 24″·786 6
$\Delta\lambda$	=	+ 0° 29′ 54″·153 6	− 0° 29′ 54″·153 6
Sin ϕ	=	− 0·610 896 049 5	− 0·615 008 470 5
Cos ϕ	=	+ 0·791 710 816 3	+ 0·788 520 501 4
Tan ϕ	=	− 0·771 615 136 4	− 0·779 952 416 5
ν	=	6 386 142·439	6 386 250·478
Sin $\Delta\lambda$	=	+ 0·008 698 192 4	− 0·008 698 192 4
Cos$\Delta\lambda$	=	+ 0·999 962 170 0	+ 0·999 962 170 0
Tan ξ_2	=	− 0·779 917 414 3	− 0·771 649 998 2
τ	=	− 0·006 596 113 4	+ 0·006 523 361 3
Tan α_{12}	=	− 1·318 684 483 2	− 1·333 391 177 4
α_{12}	=	**127° 10′ 27″·08**	**306° 52′ 07″·34**
Sin α_{12}	=	+ 0·796 802 205 0	− 0·800 012 493 8
Cos α_{12}	=	− 0·604 240 222 2	+ 0·599 983 341 2
χ	=	+ 0·010 916 375 9	
Cos ζ_2	=	+ 0·788 533 885 9	
Sin σ	=	+ 0·008 607 932 3	
σ (radians)	=	+ 0·008 608 038 6	
g	=	− 0·050 15	
h	=	− 0·039 27	
$-\sigma^2$ term	=	$-1{\cdot}902.10^{-8}$	
$+\sigma^3$ term	=	$+1{\cdot}566.10^{-10}$	
$+\sigma^4$ term	=	$-6{\cdot}109.10^{-14}$	
$-\sigma^5$ term	=	$-1{\cdot}940.10^{-15}$	
Sum	=	$-1{\cdot}886.10^{-8}$	
s	=	**54 972·161 metres**	

3.7 CONVERSION FROM THE NORMAL SECTION TO THE GEODESIC

3.7.1 Angles and azimuths are observed in normal sections. It is seldom necessary to compute geodesic azimuths, but if required they can be computed from the normal section by adding:

$-(e'^2/12)\,(s/R)^2\,(\cos^2 \phi \sin 2\alpha_{12}) + (e'^2/48)\,(s/R)^3\,(\sin \alpha_{12} \sin 2\phi)$

Any reasonable value can be used for R. At 1 500 kilometres, this correction can attain 7″, and the formula is accurate to about 0″·6. At greater distances the accuracy of the formula deteriorates, the correction reaching 1′ 31″ with errors up to 2″·5 at 4 500 kilometres, and 4′ 40″ with errors up to 44″ at 9 000 kilometres.

3.7.2 The difference in length between the normal section and the geodesic seldom attains 1 mm at 1 500 kilometres. It can usually be ignored.

3.7.3 In the examples given in paragraphs *3.4* and *3.6*, the corrections to obtain geodesic azimuths are:

		α_{12}	α_{21}
$(s/R)^2$ term	=	+ 0.″005 181	+ 0.″005 124
$(s/R)^3$ term	=	− 0.″000 014	+ 0.″000 014

4 Rigorous Formulae Between Spheroid and Grid

4.1 AIMS

4.1.1 The aims of this chapter are to provide:

4.1.1.1 The adopted formula for meridian distance;

4.1.1.2 Redfearn's formulae for obtaining easting, northing, grid convergence and point scale factor, from latitude and longitude;

4.1.1.3 Redfearn's formulae for the reverse computation, from grid to spheroid;

4.1.1.4 Numerical examples.

4.1.2 Redfearn's formulae were published in *Empire Survey Review* No. 69, 1948. They are accurate to better than 1 mm in any zone of the Australian Map Grid. For purposes of definition, they are to be regarded as exact, not merely as the opening terms of infinite series. All angles are in radians. For the definition of symbols, see paragraph *2.1.*

4.1.3 It is not foreseen that Redfearn's formulae will ever be used for hand computation. For routine work, computations using the tables published by the US Army give results which should be adequate for all practical purposes – see paragraphs *5.8* and *5.10.* If extreme precision is required for an occasional job, it is easier and better to use an electronic computer – see paragraph *1.8.3.*

4.1.4 In the numerical examples, all trigonometrical functions and intermediate results are given. They should be adequate for checking other computer programs. They have not been laid out in a form specifically designed for hand computation, and they may give a false idea of ease and brevity.

4.2 MERIDIAN DISTANCE

In Australia, meridian distance is defined by the following terms only of an infinite series:

$$\begin{aligned} m &= a(A_0\,\phi - A_2 \sin 2\phi + A_4 \sin 4\phi - A_6 \sin 6\phi) \\ \text{where } A_o &= 1 - e^2/4 - 3e^4/64 - 5e^6/256 \\ A_2 &= (3/8)\,(e^2 + e^4/4 + 15e^6/128) \\ A_4 &= (15/256)\,(e^4 + 3e^6/4) \\ A_6 &= 35e^6/3072 \end{aligned}$$

This limited formula is correct to less than 0·5 mm in latitude 45°. See paragraph *3.2.4.*

4.3 SPHEROID TO GRID: REDFEARN'S FORMULAE

4.3.1 Easting

$$\begin{aligned} E' = k_0 \Big\{ & \nu\,\omega \cos\phi \\ & + \nu \frac{\omega^3}{6} \cos^3\phi\,(\psi - t^2) \\ & + \nu \frac{\omega^5}{120} \cos^5\phi\,[4\psi^3(1 - 6t^2) + \psi^2(1 + 8t^2) - \psi(2t^2) + t^4] \\ & + \nu \frac{\omega^7}{5040} \cos^7\phi\,(61 - 479t^2 + 179t^4 - t^6)\Big\} \end{aligned}$$

4.3.2 Northing

$$\begin{aligned} N' = k_0 \Big\{ & m + \nu \sin\phi\,\frac{\omega^2}{2} \cos\phi \\ & + \nu \sin\phi\,\frac{\omega^4}{24} \cos^3\phi\,(4\psi^2 + \psi - t^2) \end{aligned}$$

$$+\nu \sin\phi \ \frac{\omega^6}{720} \ \cos^5\phi \ [8\psi^4(11-24t^2) - 28\psi^3(1-6t^2) + \psi^2(1-32t^2) - \psi(2t^2) + t^4]$$

$$+\nu \sin\phi \ \frac{\omega^8}{40320} \ \cos^7\phi \ (1385 - 3111t^2 + 543t^4 - t^6)\Big\}$$

4.3.3 Grid Convergence (radians)

$$\gamma = -\sin\phi \ \omega$$

$$-\sin\phi \ \frac{\omega^3}{3} \ \cos^2\phi \ (2\psi^2 - \psi)$$

$$-\sin\phi \ \frac{\omega^5}{15} \ \cos^4\phi \ [\psi^4(11-24t^2) - \psi^3 \ (11-36t^2) + 2\psi^2 \ (1-7t^2) + \psi t^2]$$

$$-\sin\phi \ \frac{\omega^7}{315} \ \cos^6\phi \ (17 - 26t^2 + 2t^4)$$

4.3.4 Point Scale Factor

$$k = k_0 \Big\{1 + \frac{\omega^2}{2} \ \cos^2\phi \ \psi$$

$$+ \frac{\omega^4}{24} \ \cos^4\phi \ [4\psi^3(1-6t^2) + \psi^2(1+24t^2) - 4\psi t^2]$$

$$+ \frac{\omega^6}{720} \ \cos^6\phi \ (61 - 148t^2 + 16t^4)\Big\}$$

4.4 FROM AUSTRALIAN GEODETIC DATUM TO AUSTRALIAN MAP GRID: NUMERICAL EXAMPLE

Station: BUNINYONG

Latitude ϕ = – 37° 39′ 15″·557 1
Zone: 54 λ_0 = 141°

Longitude λ = + 143° 55′ 30″·633 0
ω = + 2° 55′ 30″·633 0
ω in radians = + 0·051 053 949 6

Functions:

ϕ radians = – 0·657 191 886 3
$\sin\phi$ = – 0·610 896 049 5
$\sin 2\phi$ = – 0·967 306 020 1
$\sin 4\phi$ = – 0·490 640 893 3
$\sin 6\phi$ = + 0·718 441 150 9

e^2 = 0·006 694 541 9
e^4 = 0·000 044 816 9
e^6 = 0·000 000 300 0

A_0 = 0·998 324 257 9
A_2 = 0·002 514 668 0
A_4 = 0·000 002 639 2
A_6 = 0·000 000 003 4

Meridian Distance:

1st term = – 4 184 650·835
2nd term = + 15 514·577
3rd term = – 8·259
4th term = – 0·016
Sum = m = – 4 169 144·533

Radii of Curvature:

ρ = 6 359 277·924
ν = 6 386 142·439

Powers:

Power	$\cos\phi$	ω	t= $\tan\phi$	$\psi = \nu/\rho$
1	0·791 710 816 3	0·051 053 949 6	– 0·771 615 136 4	1·004 224 459 9
2	0·626 806 016 6	0·002 606 505 8	0·595 389 918 7	1·008 466 765 9
3	0·496 249 103 1	0·000 133 072 4		1·012 726 993 3
4	0·392 885 782 5	0·000 006 793 9	0·354 489 155 3	1·017 005 217 8
5	0·311 051 923 6	0·000 000 346 9		
6	0·246 263 172 3	0·000 000 017 7	0·211 059 269 4	
7	0·194 969 217 2	0·000 000 000 9		
8		(+4·6.10^{-11})		

Easting:

1st term = + 258 127·648
2nd term = + 28·736
3rd term = – 0·031
4th term = $(-3{\cdot}6.10^{-5})$
Sum = + 258 156·353
Sum. k_0 = E′ = + 258 053·090
False origin = + 500 000·000
E = + 758 053·090

Northing:

m = – 4 169 144·533
1st term = – 4 025·327
2nd term = – 2·435
3rd term = – 0·001
4th term = $(+\,2{\cdot}4.10^{-7})$
Sum = – 4 173 172·295
Sum.k_0 = N′ = – 4 171 503·026
False origin = + 10 000 000·000
N = + 5 828 496·974

Grid Convergence:

1st term = $+\,1^{\circ}\,47'\,13''.12$
2nd term = $+\,3''.55$
3rd term = $(+\,2{\cdot}0.10^{-3})$
4th term = $(+\,2{\cdot}0.10^{-7})$
Sum = γ = $+\,1^{\circ}\,47'\,16''.67$

Point Scale:

1st term = + 1·000 820 34
2nd term = + 0·000 000 29
3rd term = $(-\,1{\cdot}3.10^{-10})$
Sum = + 1·000 820 63
Sum. k_0 = k = + 1·000 420 30

4.5 GRID TO SPHEROID: REDFEARN'S FORMULAE

4.5.1 **Latitude** (radians)

$$\begin{aligned}\phi = \phi' &- (t'/k_0\rho')(E'^2/2k_0\nu')\\ &+ (t'/k_0\rho')(E'^4/24k_0^3\nu'^3)\,[-4\psi'^2 + 9\psi'(1-t'^2) + 12t'^2]\\ &- (t'/k_0\rho')(E'^6/720k_0^5\nu'^5)\,[8\psi'^4(11-24t'^2) - 12\psi'^3(21-71t'^2)\\ &+ 15\psi'^2(15-98t'^2+15t'^4) + 180\psi'(5t'^2-3t'^4) + 360t'^4]\\ &+ (t'/k_0\rho')(E'^8/40320k_0^7\nu'^7)(1385 + 3633t'^2 + 4095t'^4 + 1575t'^6)\end{aligned}$$

4.5.2 **Longitude** (radians)

$$\begin{aligned}\omega = {} & \sec\phi'\,(E'/k_0\nu')\\ &- \sec\phi'\,(E'^3/6k_0^3\nu'^3)(\psi' + 2t'^2)\\ &+ \sec\phi'\,(E'^5/120k_0^5\nu'^5)\,[-4\psi'^3(1-6t'^2) + \psi'^2(9-68t'^2) + 72\psi't'^2 + 24t'^4]\\ &- \sec\phi'\,(E'^7/5040k_0^7\nu'^7)(61 + 662t'^2 + 1320t'^4 + 720t'^6)\end{aligned}$$

4.5.3 **Grid Convergence** (radians)

$$\begin{aligned}\gamma = {} & -t'(E'/k_0\nu')\\ &+ t'(E'^3/3k_0^3\nu'^3)(-2\psi'^2 + 3\psi' + t'^2)\\ &- t'(E'^5/15k_0^5\nu'^5)\,[\psi'^4(11-24t'^2) - 3\psi'^3(8-23t'^2) + 5\psi'^2(3-14t'^2) + 30\psi't'^2 + 3t'^4]\\ &+ t'(E'^7/315k_0^7\nu'^7)(17 + 77t'^2 + 105t'^4 + 45t'^6)\end{aligned}$$

4.5.4 **Point Scale Factor**

$$\begin{aligned}k = k_0\,\{&1 + E'^2/2k_0^2\rho'\nu'\\ &+ (E'^4/24k_0^4\rho'^2\nu'^2)\,[4\psi'(1-6t'^2) - 3(1-16t'^2) - 24t'^2/\psi']\\ &+ (E'^6/720k_0^6\rho'^3\nu'^3)\}\end{aligned}$$

4.6 FROM AUSTRALIAN MAP GRID TO AUSTRALIAN GEODETIC DATUM: NUMERICAL EXAMPLE

Station: BUNINYONG

E = 758 053·090
False origin = – 500 000·000
E′ = + 258 053·090
E′/k_0 = + 258 156·353

Zone: 54; k_0 = 0·999 6

N = 5 828 496·974
False origin = – 10 000 000·000
N′ = – 4 171 503·026
N′/k_0 = – 4 173 172·295

Latitude ϕ' for which the meridian distance equals N′/k_0, by iteration or interpolation:

ϕ' = $-\,37^{\circ}\,41'\,26''.198\,2$
sec ϕ' = 1·263 705 299 7

ρ' = 6 359 317·153
ν' = 6 386 155·570

Powers of functions of the latitude ϕ':

x	$E'^x/k_0^x\rho'\nu'^{(x-1)}$	$E'^x/k_0^x\nu'^x$	$E'^x/k_0^x(\rho'\nu')^{x/2}$	$t^x = \tan^x \phi'$	$\psi^x = (\nu'/\rho')^x$
1		0·040 424 375 8		– 0·772 626 096 2	1·004 220 330 1
2	0·001 641 026 7		0·001 641 026 7	+ 0·596 951 084 5	1·008 458 471 4
3		0·000 066 058 7			1·012 714 499 0
4	0·000 002 681 7		0·000 002 693 0	+ 0·356 350 597 3	1·016 988 488 5
5		0·000 000 107 9			
6	0·000 000 004 4		0·000 000 004 4	+ 0·212 723 875 6	
7		0·000 000 000 2			
8	$(+7{\cdot}2.10^{-12})$				

Latitude:

ϕ'	=	–	37° 41′ 26″198 2
1st term	=	+	2 10 ·761 6
2nd term	=	–	0 ·120 6
3rd term	=	+	0 ·000 1
4th term	=		$(-1{\cdot}5.10^{-7})$
Sum = ϕ	=	–	37° 39′ 15″557 1

Longitude:

λ_0	=	+	141° 00′ 00″000 0
1st term	=	+	2 55 36 ·934 1
2nd term	=	–	6 ·308 1
3rd term	=	+	0 ·007 1
4th term	=		$(-9{\cdot}9.10^{-6})$
Sum = λ	=	+	143° 55′ 30″633 0

Grid Convergence:

1st term	=	+	1° 47′ 22″25
2nd term	=	–	5 ·59
3rd term	=	+	0 ·01
4th term	=		$(-9{\cdot}8.10^{-6})$
Sum = γ	=	+	1° 47′ 16″67

Point Scale:

1st term	=	+	1·000 820 51
2nd term	=	+	0·000 000 11
3rd term	=		$(+6{\cdot}1.10^{-12})$
Sum	=	+	1·000 820 63
Sum.k_0 = k	=	+	1·000 420 30

5 First Order Formulae

5.1 AIM

The aim of this chapter is to provide working formulae of first order accuracy, for use with desk calculators and natural tables. Computation forms and examples are included. For a list of the computations included in the chapter, see the table of contents at the beginning of the manual.

5.2 TABLES

5.2.1 For first order work, seven-figure trigonometrical tables suffice, but eight-figure tables are more widely available, and easy to use.

5.2.2 In addition to tables of trigonometrical functions, the first three volumes of the US Army Tables are required – see paragraph *1.7.*

5.3 INTERPOLATION IN TABLES

5.3.1 Functions in the US Army Tables are given at intervals of 1′ of latitude, and differences are given for 1″. Interpolation should be done in a self-checking manner. A method which can be modified to suit most machines follows:

Latitude 37° 39′ 15″·557 1	To find (B) (Latitude Functions page 59)	To find (I) (Geographic to Grid, page 59)
Put the tabulated value for 37° 39′ into the product register followed by at least six zeros: Clear the other registers.	324·352 796	4 166 997·431
Enter on the keyboard the tabulated difference for 1″, taking care with the decimal. The differences are usually given to two more places than the main entry:	15·31	30.818 34
Multiply positively or negatively by 15·557 1 and record the result:	324·352 558	4 167 476·875
Clear the counter register only, and multiply in the same sense by the complement of 15·557 1, 44·442 9 which can be determined mentally. Check that the next tabulated value is obtained, within 1 in the last place:	324·351 877	4 168 846·531

The unchanging part of the main entries, 32 and 416, may have to be omitted from machines with small registers.

The practice of calculating increments separately, recording them on paper, and adding them by hand, is to be avoided.

5.3.2 REVERSE INTERPOLATION

The reverse process is also required. When converting grid coordinates to geographicals in paragraph *5.10.4,* it is necessary to find the latitude ϕ' corresponding to a given value of N′ by a reverse interpolation in table (I). The figures in this example are from the computation in paragraph *5.11.*

$N' = 4\ 171\ 503{\cdot}026$

	To find ϕ' (Grid to Geographic, page 59)
Enter N′ on the left end of the product register:	4 171 503·026
Enter the preceding tabulated value from table (I) on the keyboard, and note 37° 41′:	4 170 695·637
Subtract:	807·389
Enter the difference for 1″ on the keyboard, with the decimal in the right place:	30·818 51
Clear the counter register, and divide:	26·198 2
Answer:	37° 41′ 26″·198 2
To check, clear the machine, and enter the subsequent value for (I) in the left end of the product register. Note 37° 42′:	4 172 544·748
Enter N′ on the keyboard:	4 171 503·026
Subtract:	1 041·722
Enter the difference for 1″ with the decimal in the right place:	30·818 51
Clear the counter register and divide:	33·801 8
By inspection, subtract from 60 and check:	37° 41′ 26″·198 2

5.4 LATITUDE AND LONGITUDE FROM SPHEROIDAL DISTANCE AND AZIMUTH: PUISSANT'S FORMULAE

5.4.1 The following version of Puissant's formulae makes use of the functions A, B, C, D, E, F tabulated on pages 2-181 of the tables of Latitude Functions, and the arc/sine ratios on pages 272-273. The formulae are accurate to one part per million (ppm) at distances of 80 kilometres, and are convenient for desk calculators. In addition to the latitude and longitude of the distant point, the reverse azimuth is calculated.

5.4.2 See the computation form and the example. The formulae below are given with the correct signs. It is, however, easy to make a mistake in carrying the signs through the algebra, as the signs of C, D and F are negative south of the equator. In practice it is best to determine the signs of each individual term from the diagrams on the computation form.

5.4.2.1 Enter the data on the form, look up $\sin\alpha_1$ and $\cos\alpha_1$ and B, C, D, E for ϕ_1.

5.4.2.2 First find an approximate value $\Delta'\phi$ for the difference in latitude from the formula

$$\Delta'\phi'' = B_1\ s\cos\alpha_1 - C_1\ s^2\ \sin^2\alpha_1 - (B_1\ s\cos\alpha_1)\ E_1\ s^2\ \sin^2\alpha_1$$

Then the true difference in latitude is

$$\Delta\phi'' = \Delta'\phi'' - D_1\ (\Delta'\phi'')^2$$

5.4.2.3 For the difference in longitude,

$$\sin\Delta\lambda = \sin(A_2\ s)\sin\alpha_1/\cos\phi_2$$

As $\Delta\lambda$ and (A s) are small angles, instead of looking up the sines in trigonometrical tables it is easier and quicker to use the tables of arc/sine ratios on pages 272-3 of the tables of Latitude Functions. The table is entered from the $\Delta\lambda''$ column with the entry on line 27 of the computation form, and the interpolated value written on line 28; and the table is entered again with s from line 4, and the interpolated value written on line 29. Note that A is for ϕ_2.

5.4.2.4 For azimuth,

$$\Delta\alpha'' = \Delta\lambda'' \sin \phi_m / \cos (\Delta\phi/2) + F_m (\Delta\lambda'')^3$$

Note that F is for ϕ_m.

LATITUDE AND LONGITUDE FROM DISTANCE AND AZIMUTH

PUISSANT'S FORMULAE

From STATION 1 ______________ To STATION 2 ______________

1	ϕ_1	
2	λ_1	
3	α_1	
4	s (metres)	

$$\Delta'\phi'' = B_1.s.\cos\alpha_1 \ - C_1.s^2.\sin^2\alpha_1 \ -(B_1.s.\cos\alpha_1)\ E_1.s^2.\sin^2\alpha_1$$

$$\Delta\phi'' = \Delta'\phi'' - D_1.\ (\Delta\phi'')^2$$

$$\sin\Delta\lambda = \sin(A_2.s).\sin\alpha_1/\cos\phi_2$$

$$\Delta\alpha'' = [\Delta\lambda''.\sin\phi_m/\cos(\Delta\phi/2)] + F_m.(\Delta\lambda'')^3$$

5	$s.10^{-4}$	
6	$\cos\alpha_1$	
7	$B.10^4$ for ϕ_1	
8	$\sin\alpha_1$	
9	5 . 8	
10	$C.10^8$ for ϕ_1	
11	$D.10^8$ for ϕ_1	
12	$E.10^{12}$ for ϕ_1	

13		5 . 6 . 7	±	
14		9^2 . 10	±	
15		9^2 . 12 . 13 . 10^{-4}	±	
16		$13+14+15=\Delta'\phi''$		
17		$[16 . 10^{-4}]^2 . 11$	±	
18		$16+17=\Delta\phi''$		
19		$\Delta\phi$	±	
20		$\Delta\phi/2$		
21		ϕ_1	±	
22		ϕ_m		
23		$\Delta\phi/2$		
24	LATITUDE	ϕ_2	±	

25	$A.10^4$ for ϕ_2	
26	$\cos\phi_2$	
27	25 . 9 / 26 ≈ $\Delta\lambda''$	
28	arc/sin 27 (p272: $\Delta\lambda''$)	
29	arc/sin 4 (p272:s)	

30		$27 . 28/29 = \Delta\lambda''$		
31		$\Delta\lambda$	±	
32		λ_1		
33	LONGITUDE	λ_2		

34	sin 22	
35	cos 20	
36	$[30 .10^{-4}]^3$	
37	$F.10^{12}$ for ϕ_m	

38		30 . 34/35		
39		36 . 37	±	
40		$38+39=\Delta\alpha''$		
41		$\Delta\alpha$	±	
42		$\alpha_1 \pm 180°$		
43	AZIMUTH	α_2		

SIGN CONVENTION

Latitude ϕ : North +
South −
Longitude λ : East +
West −

Determine the signs of each term from the diagram below and form sums ignoring signs in the formulae above

NORTHERN HEMISPHERE

14 & 17 are −

$\alpha_1 = 0°$

13	+	+
15	−	−
39	−	+
$\Delta\phi$	+	+
$\Delta\lambda$	−	+
$\Delta\alpha$	−	+
13	−	−
15	+	+
39	−	−
$\Delta\phi$	−	−
$\Delta\lambda$	−	+
$\Delta\alpha$	−	+

$\alpha_1 = 180°$

SOUTHERN HEMISPHERE

14 & 17 are +

$\alpha_1 = 0°$

13	+	+
15	−	−
39	+	−
$\Delta\phi$	+	+
$\Delta\lambda$	−	+
$\Delta\alpha$	+	−
13	−	−
15	+	+
39	+	−
$\Delta\phi$	−	−
$\Delta\lambda$	−	+
$\Delta\alpha$	+	−

$\alpha_1 = 180°$

Computed ______________ checked ______________ Date ______________

5.5

LATITUDE AND LONGITUDE FROM DISTANCE AND AZIMUTH

PUISSANT'S FORMULAE

From STATION 1 BUNINYONG To STATION 2 FLINDERS PEAK

1	ϕ_1	37 39 15·5571	
2	λ_1	143 55 30·6330	
3	α_1	127 10 27·08	
4	s (metres)	54972·161	
5	$s.10^{-4}$	5·497 2161	
6	$\cos\alpha_1$	0·604 24023	
7	$B.10^4$ for ϕ_1	324·352 558	
8	$\sin\alpha_1$	0·796 80221	
9	5 . 8	4·380 1939	
10	$C.10^8$ for ϕ_1	0·195 9519	
11	$D.10^8$ for ϕ_1	2·360 5	
12	$E.10^{12}$ for ϕ_1	0·011 38	
13		5 . 6 . 7 ±	− 1077·3821
14		9^2 . 10 ±	+ 3·7596
15		9^2 . 12 . 13 . 10^{-4} ±	+ 0·0235
16		$13+14+15=\Delta'\phi''$	− 1073·5990
17		$[16 . 10^{-4}]^2 . 11$ ±	+ 0·0272
18		$16+17=\Delta\phi''$	− 1073·5718
19		$\Delta\phi$ ±	−00 17 53·5718
20		$\Delta\phi/2$	−00 08 56·7859
21		ϕ_1 ±	−37 39 15·5571
22		ϕ_m	−37 48 12·3430
23		$\Delta\phi/2$	−00 08 56·7859
24	LATITUDE	ϕ_2 ±	−37° 57′ 09″1289
25	$A.10^4$ for ϕ_2	322·982 643	
26	$\cos\phi_2$	0·788 52050	
27	25 . 9 / 26 ≈ $\Delta\lambda''$	1794·153 2	
28	arc/sin 27 (p272: $\Delta\lambda''$)	1·000 01260	
29	arc/sin 4 (p272:s)	1·000 01234	
30		$27 . 28/29=\Delta\lambda''$	1794·1537
31		$\Delta\lambda$ ±	+00 29 54·1537
32		λ_1	+143 55 30·6330
33	LONGITUDE	λ_2	+144° 25′ 24″·7867
34	sin 22	0·612 95433	
35	cos 20	0·999 99661	
36	$[30 . 10^{-4}]^3$	0·005 77536	
37	$F.10^{12}$ for ϕ_m	0·749 5	
38		30 . 34 / 35	1099·74
39		36 . 37 ±	0·00
40		$38+39=\Delta\alpha''$	1099·74
41		$\Delta\alpha$ ±	− 00 18 19·74
42		$\alpha_1 \pm 180°$	307 10 27·08
43	AZIMUTH	α_2	306°52′ 07″34

$$\Delta'\phi'' = B_1.s.\cos\alpha_1 \; - C_1.s^2.\sin^2\alpha_1 \; -(B_1.s.\cos\alpha_1)\; E_1.s^2.\sin^2\alpha_1$$

$$\Delta\phi'' = \Delta'\phi'' - D_1.\,(\Delta\phi'')^2$$

$$\sin\Delta\lambda = \sin(A_2.s).\sin\alpha_1/\cos\phi_2$$

$$\Delta\alpha'' = [\Delta\lambda''.\sin\phi_m/\cos(\Delta\phi/2)] + F_m.(\Delta\lambda'')^3$$

SIGN CONVENTION

Latitude ϕ : North +
South −

Longitude λ : East +
West −

Determine the signs of each term from the diagram below and form sums ignoring signs in the formulae above

NORTHERN HEMISPHERE

14 & 17 are −

$\alpha_1 = 0°$

13	+	+
15	−	−
39	−	+
$\Delta\phi$	+	+
$\Delta\lambda$	−	+
$\Delta\alpha$	−	+
13	−	−
15	+	+
39	−	−
$\Delta\phi$	−	−
$\Delta\lambda$	−	+
$\Delta\alpha$	−	+

$\alpha_1 = 180°$

SOUTHERN HEMISPHERE

14 & 17 are +

$\alpha_1 = 0°$

13	+	+
15	−	−
39	+	−
$\Delta\phi$	+	+
$\Delta\lambda$	−	+
$\Delta\alpha$	+	−
13	−	−
15	+	+
39	+	−
$\Delta\phi$	−	−
$\Delta\lambda$	−	+
$\Delta\alpha$	+	−

$\alpha_1 = 180°$

Computed R. [illegible] checked M.J [illegible] Date Tues 11 Feb 69

5.6 DISTANCE AND AZIMUTHS FROM LATITUDE AND LONGITUDE: GAUSS MID-LATITUDE FORMULAE

5.6.1 The following version of the Gauss Mid-Latitude formulae makes use of the functions A, B and F tabulated on pages 2-181 of the tables of Latitude Functions. Natural values of ν, the radius of curvature in the prime vertical, on pages 182-271, and of the arc/sine ratios on pages 272-273, are also used. The formulae are accurate to one ppm at distances of 100 kilometres, and are convenient for desk calculators.

5.6.2 See the computation form and the example. The formulae given below have the correct signs applicable in either hemisphere but in practice it is easier to take the sign from the diagram on the computation forms.

5.6.2.1 Enter the data on the form, and calculate ϕ_m, $\Delta\phi$, $\phi/2$, $\Delta\lambda$ and $\Delta\lambda/2$. Note that $\Delta\phi = \phi_2 - \phi_1$ and $\Delta\lambda = \lambda_2 - \lambda_1$, following the convention used throughout this manual. Look up cos $(\Delta\phi/2)$ and cos $(\Delta\lambda/2)$; sin ϕ_m and cos ϕ_m; and A, B, F and ν for ϕ_m.

5.6.2.2 Calculate $\Delta'\phi$ and $\Delta'\lambda$ from the formulae

$$\Delta'\phi'' = \Delta\phi''/\ [\text{arc/sin}\ (\Delta\phi/2)]\ ;\ \Delta'\lambda'' = \Delta\lambda''/[\text{arc/sin}\ (\Delta\lambda/2)]$$

using the tables of arc/sine ratios on page 272 of the tables of Latitude Functions, entering the table in both cases from the column headed $\Delta\lambda''$.

5.6.2.3 Calculate $\Delta\alpha$ from the formula

$$\Delta\alpha'' = [\Delta\lambda''\ \sin\phi_m/\cos\ (\Delta\phi/2)]\ +\ F_m\ (\Delta\lambda'')^3$$

5.6.2.4 Calculate

$$s'\ \sin\ (\alpha + \Delta\alpha/2) = (\Delta'\lambda\ \cos\ \phi_m)/A_m \text{ and}$$

$$s'\ \cos\ (\alpha + \Delta\alpha/2) = [\Delta'\phi\ \cos\ (\Delta\lambda/2)]\ /B_m$$

Divide the smaller by the larger and determine $(\alpha + \Delta\alpha/2)$ from the tangent (if sin/cos) or from the cotangent (if cos/sin). Then

$$\alpha_1 = (\alpha + \Delta\alpha/2) - \Delta\alpha/2 \text{ and } \alpha_2 = (\alpha + \Delta\alpha/2) + \Delta\alpha/2$$

5.6.2.5 Look up the sine or cosine, whichever is the larger, of $(\alpha + \Delta\alpha/2)$ and calculate s′ from one of the formulae in *5.6.2.4* above.

5.6.2.6 Calculate the distance from

$$s = s'\ \text{arc/sin}\ (s'/2\nu_m)$$

entering the arc/sine tables on page 272 from the column headed s. Note that ν is for ϕ_m

DISTANCE AND AZIMUTH FROM LATITUDE AND LONGITUDE

GAUSS MID-LATITUDE FORMULAE

STATION 1 ____________ ____________ ______

STATION 2 ____________ ____________ ______

1	ϕ_1	
2	ϕ_2	
3	$\phi_2 - \phi_1 = \Delta\phi$ ±	
4	$\Delta\phi/2$	
5	$\phi_1 + \Delta\phi/2 = \phi_m$	
6	$\phi_m + \Delta\phi/2 = \phi_2$	
7	$\Delta\phi''$	
8	$\Delta\phi''/2$	

9	λ_1	
10	λ_2	
11	$\lambda_2 - \lambda_1 = \Delta\lambda$ ±	
12	$\Delta\lambda/2$	
13	$\Delta\lambda''$	
14	$\Delta\lambda''/2$	

15	arc/sin 8 (p272: $\Delta\lambda'$)	
16	$7/15 = \Delta'\phi''$	

17	arc/sin 14 (p272: $\Delta\lambda'$)	
18	$13/17 = \Delta'\lambda''$	

19	$\sin\phi_m$	
20	cos 4	
21	13.19/20	
22	F.10^{12} for ϕ_m	
23	$22.(13.10^{-4})^3$	
24	$21+23=\Delta\alpha''$	
25	$\Delta\alpha$ ±	

39	si n 33	
40	(18.26)/(30.39)	
	OR(select the larger)	
39	cos 33	
40	(16.29)/(27.39)	
41	$40.10^4 = s'$	

26	$\cos\phi_m$	
27	B.10^4 for ϕ_m	
28	26.27.18	
29	cos 12	
30	A.10^4 for ϕ_m	
31	29.30.16	
32	$28/31 = \tan(\alpha+\Delta\alpha/2)$	
	OR (select the larger)	
32	$31/28 = \cot(\alpha+\Delta\alpha/2)$	
33	$\alpha+\Delta\alpha/2$	
34	$-25/2$ ±	
35	**Azimuth** α_1	

42	ν = N for ϕ_m (p182-270)	
43	$42.2.10^{-5}$	
44	43.0·48481	
45	41/44	
46	arc/sin 45 (p272: $\Delta\lambda''$)	
47	41.46 = **Distance s**	

36	$35 \pm 180°$	
37	$+25$ ±	
38	**Reverse Azimuth** α_2	

$$\Delta'\phi = \Delta\phi''/\mathrm{arc/sin}(\Delta\phi/2)$$

$$\Delta'\lambda = \Delta\lambda''/\mathrm{arc/sin}(\Delta\lambda/2)$$

$$\Delta\alpha'' = [\Delta\lambda''.\sin\phi_m/\cos(\Delta\phi/2)] + F_m(\Delta\lambda'')^3$$

$$s'.\sin(\alpha+\Delta\alpha/2) = (\Delta'\lambda.\cos\phi_m)/A_m$$

$$s'.\cos(\alpha+\Delta\alpha/2) = [\Delta'\phi.\cos(\Delta\lambda/2)]/B_m$$

$$s = s'.\mathrm{arc/sin}(s'/2\nu_m)$$

SIGN CONVENTION

Latitude ϕ : NORTH +
: SOUTH −

Longitude λ : EAST +
: WEST −

NORTHERN HEMISPHERE

When STATION 2 is $\frac{\text{EAST}}{\text{WEST}}$ of STATION 1 $\Delta\alpha$ is $\frac{+}{-}$

SOUTHERN HEMISPHERE

When STATION 2 is $\frac{\text{EAST}}{\text{WEST}}$ of STATION 1 $\Delta\alpha$ is $\frac{-}{+}$

Computed ______ ______ ______ ______ ______

Checked ______ ______ ______ ______ ______

Date ______ ______ ______ ______ ______

DISTANCE AND AZIMUTH FROM LATITUDE AND LONGITUDE

GAUSS MID-LATITUDE FORMULAE

STATION 1 BUNINYONG

STATION 2 FLINDERS PEAK

1	ϕ_1	-37 39 15·5571	9	λ_1	+143 55 30·6330
2	ϕ_2	-37 57 09·1288	10	λ_2	+144 25 24·7866
3	$\phi_2 - \phi_1 = \Delta\phi$ ±	- 17 53·5717	11	$\lambda_2 - \lambda_1 = \Delta\lambda$ ±	+ 29 54·1536
4	$\Delta\phi/2$	- 08 56·7858	12	$\Delta\lambda/2$	+ 14 57·0768
5	$\phi_1 + \Delta\phi/2 = \phi_m$	-37 48 12·3429	13	$\Delta\lambda''$	1794·1536
6	$\phi_m + \Delta\phi/2 = \phi_2$	-37 57 09·1288	14	$\Delta\lambda''/2$	897·1
7	$\Delta\phi''$	1073·5717			
8	$\Delta\phi''/2$	536·8			

15	arc/sin 8 (p272: $\Delta\lambda''$)	1·000 00113	17	arc/sin 14 (p272: $\Delta\lambda'$)	1·000 00315
16	$7/15 = \Delta'\phi''$	1073·5705	18	$13/17 = \Delta'\lambda''$	1794·1479

19	$\sin\phi_m$	0·612 95433	39	sin 33	0·798 41015
20	cos 4	0·999 99661	40	(18.26)/(30.39)	5·497 1991
21	13.19/20	1099·738		OR (select the larger)	
22	$F.10^{12}$ for ϕ_m	0·7495	39	cos 33	
23	$22.(13.10^{-4})^3$	0·004	40	(16.29)/(27.39)	
24	$21+23=\Delta\alpha''$	1099·742			
25	$\Delta\alpha$ ±	-00 18 19·74	41	$40.10^4 = s'$	54 971·991

26	$\cos\phi_m$	0·790 11833	42	ν = N for ϕ_m (p182-270)	6 386 196·421
27	$B.10^4$ for ϕ_m	324·344 333	43	$42.2.10^{-5}$	127·72392
28	26.27.18	459 787·0048	44	43.0·48481	61·9218
29	cos 12	0·999 99054	45	41/44	887·8
30	$A.10^4$ for ϕ_m	322·985 378			
31	29.30.16	346 744·2936	46	arc/sin 45 (p272: $\Delta\lambda''$)	1·000 00309
32	$28/31 = \tan(\alpha+\Delta\alpha/2)$		47	41.46 = Distance s	54 972·161
	OR (select the larger)				
32	$31/28 = \cot(\alpha+\Delta\alpha/2)$	0·754 1411			
33	$\alpha+\Delta\alpha/2$	127 01 17·21			
34	$-25/2$ ±	+00 09 09·87			
35	Azimuth α_1	127° 10' 27''·08			

36	$35 \pm 180°$	307 10 27·08
37	$+25$ ±	-00 18 19·74
38	Reverse Azimuth α_2	306 52 07·34

$$\Delta'\phi = \Delta\phi''/\text{arc/sin}(\Delta\phi/2)$$

$$\Delta'\lambda = \Delta\lambda''/\ \text{arc/sin}(\Delta\lambda/2)$$

$$\Delta\alpha'' = [\Delta\lambda''.\sin\phi_m/\cos(\Delta\phi/2)] + F_m(\Delta\lambda'')^3$$

$$s'.\sin(\alpha+\Delta\alpha/2) = (\Delta'\lambda.\cos\phi_m)/A_m$$

$$s'.\cos(\alpha+\Delta\alpha/2) = [\Delta'\phi.\cos(\Delta\lambda/2)]/B_m$$

$$s = s'.\ \text{arc/sin}(s'/2\,\nu_m)$$

SIGN CONVENTION

Latitude ϕ : NORTH +
: SOUTH −

Longitude λ : EAST +
: WEST −

NORTHERN HEMISPHERE

When STATION 2 is EAST/WEST of STATION 1 $\Delta\alpha$ is $\pm$

SOUTHERN HEMISPHERE

When STATION 2 is EAST/~~WEST~~ of STATION 1 $\Delta\alpha$ is $\mp$

Computed R. d'Al... WO II

Checked M. I... SGT

Date TUES 11 FEB '69

5.8 SPHEROID TO GRID, USING US ARMY TABLES

5.8.1 This computation uses Volume 1 of the US Army Tables, TM 5-241-32/1. Given the latitude and longitude of a point, its zone number, easting, northing and grid convergence can be determined. The formulae used in the tables are not quite as accurate as Redfearn's formulae given in Chapter 4, but suffice for all practical purposes. At the extreme limit of a zone overlap 3½° from the central meridian, the error will not exceed 30 mm in position, and 0″01 in grid convergence.

5.8.2 See the computation form and example at *5.9*. The zone number and central meridian can be determined at once from the table on the form.

5.8.3 The sign convention is unusual. Compute $\omega = \lambda - \lambda_0$ and $p = \omega'' 10^{-4}$. Then ω and p are always taken positive, and

$$E' = (IV)\ p + (V)\ p^3 + B_5$$

so that the sign of each term is the same as the sign of the tabulated function. (IV) is always positive, but for accurate work must include the small correction Δ^2 (IV) taken from the diagram at the bottom right corner of each page. (V) is negative between latitudes 45° 03′ and 80°. B_5 is negative between 27° 57′ and 76° 40′. Interpolate in the tables in a self-checking manner as described in paragraph *5.3.1*. The tabular values and increments need not then be recorded. West of a central meridian, E′ is taken negative, and then E = E′ + 500 000 metres.

5.8.4 Similarly,

$$N' = (I) + (II)\ p^2 + (III)\ p^4 + A_6$$

(I) and (II) are always positive, but (III) is negative beyond 65° 56′ and A_6 is negative between latitudes 50° and 68°. South of the equator, N′ is taken negative, and N = N′ + 10 000 000 metres. North of the equator, N = N′.

5.8.5 For grid convergence

$$\gamma = (XII)\ p + (XIII)\ p^3 + C_5$$

and (XII), (XIII) and C_5 are always positive. The sign of γ is taken from the diagram on the form. The sign of γ used in Australia is the opposite to C used in the US Army Tables.

5.9

TRANSFORMATION OF COORDINATES FROM GEOGRAPHIC TO GRID

AUSTRALIAN MAP GRID

ZONE ________ **STATION** ______ ______ ______ ______

Zone	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Central Meridian	87°	93°	99°	105°	111°	117°	123°	129°	135°	141°	147°	153°	159°	165°	171°	177°E

1	Latitude ϕ		
2	Longitude λ		
3	Central Meridian λ_0		
4	$\omega = \lambda - \lambda_0$		
5	ω''		
6	(I) Tabular value		
7	Increment		
8	(I) for ϕ		
9	(II) Tabular value		
10	Increment		
11	(II) for ϕ		
12	(III) for ϕ	±	
13	(IV) Tabular value		
14	Increment		
15	Δ^2 (IV)		
16	(IV) for ϕ		
17	(V) Tabular value		
18	Increment		
19	(V) for ϕ	±	
20	(XII) Tabular value		
21	Increment		
22	(XII) for ϕ		
23	(XIII) for ϕ		

24	$p = \cdot 0001 . \omega''$		+
25	p^2		+
26	p^3		+
27	p^4		+
28	16.24		+
29	19.26	±	
30	B_5 for ϕ, ω	±	
31	28 + 29 + 30 = E′	±	
32	False Origin		+ 500 000 ·000
33	31 + 32 = **Easting** E		
34	8		+
35	11.25		+
36	12.27	±	
37	A_6 for ω	±	
38	34+35+36+37 = N′	±	
39	Southern Hemisphere		+ 10 000 000 ·000
40	38+39 = **Northing** N		
41	22.24		+
42	23.26		+
43	C_5 for ω		+
44	$41 + 42 + 43 = \gamma''$		
45	**Grid Convergence** γ	±	

$E' = IV.p + V.p^3 + B_5$

$E = 500{,}000 + E'$

$N' = I + II.p^2 + III.p^4 + A_6$

N = N′ north of equator

N = 10,000,000 + N′ south of equator

$\gamma = XII.p + XIII.p^3 + C_5$

SIGN CONVENTION

NOTE: 1. p & ω are always taken positive

2. (III) is negative beyond latitude 65°56′ N & S

(V) is negative between latitudes 45°03′ & 80° N & S

B_5 is negative between latitudes 27°57′ & 76°40′ N & S

A_6 is negative between latitudes 50° & 68° N & S

3. $\gamma = -C$ used in US Army tables

Grid bearing = Azimuth + γ = Azimuth − C

Computed ____ ____ ____ ____

Checked ____ ____ ____ ____

Date ____ ____ ____ ____

NORTHERN HEMISPHERE λ_0			**SOUTHERN HEMISPHERE** λ_0	
−	+	E′	−	+
+	+	N′	−	−
+	−	γ	−	+

TRANSFORMATION OF COORDINATES FROM GEOGRAPHIC TO GRID

AUSTRALIAN MAP GRID

ZONE 55

STATION BUNINYONG.

Zone	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Central Meridian	87°	93°	99°	105°	111°	117°	123°	129°	135°	141°	147°	153°	159°	165°	171°	177°E

1	Latitude ϕ		37 39 15·5571
2	Longitude λ		143 55 30·6330
3	Central Meridian λ_0		147
4	$\omega = \lambda - \lambda_0$		- 03 04 29·3670
5	ω''		11069·3670
6	(I) Tabular value		4 166 997·431
7	Increment		+ 479·444
8	(I) for ϕ		4 167 476·875
9	(II) Tabular value		3 628·283
10	Increment		+ 0·144
11	(II) for ϕ		3 628·427
12	(III) for ϕ	±	1·979
13	(IV) Tabular value		245 036·883
14	Increment		- 14·201
15	Δ^2 (IV)		+ 0·002
16	(IV) for ϕ		245 022·684
17	(V) Tabular value		24·613
18	Increment		- 0·016
19	(V) for ϕ	±	24·597
20	(XII) Tabular value		6 108·363
21	Increment		+ 0·597
22	(XII) for ϕ		6 108·960
23	(XIII) for ϕ		3·038

24	$p = \cdot0001.\omega''$	+	1·106 93670
25	p^2	+	1·225 3089
26	p^3	+	1·356 34
27	p^4	+	1·501 4
28	16.24	+	271 224·601
29	19.26 ±	+	33·362
30	B_5 for ϕ, ω ±	-	0·040
31	28 + 29 + 30 = E′ ±	-	271 257·923
32	False Origin	+	500 000 ·000
33	31 + 32 = Easting E		228 742·077
34	8	+	4 167 476·875
35	11.25	+	4 445·944
36	12.27 ±	+	2·971
37	A_6 for ω ±	+	0·001
38	34+35+36+37 = N′ ±	-	4 171 925·791
39	Southern Hemisphere	+	10 000 000 ·000
40	38+39 = Northing N		5 828 074·209
41	22.24	+	6 762·23
42	23.26	+	4·12
43	C_5 for ω	+	0·00
44	$41 + 42 + 43 = \gamma''$	-	6 766·35
45	Grid Convergence γ ±		- 01° 52′ 46″·35

$E' = IV.p + V.p^3 + B_5$

$E = 500{,}000 + E'$

$N' = I + II.p^2 + III.p^4 + A_6$

$N = N'$ north of equator

$N = 10{,}000{,}000 + N'$ south of equator

$\gamma = XII.p + XIII.p^3 + C_5$

SIGN CONVENTION

NOTE: 1. p & ω are always taken positive

2. (III) is negative beyond latitude 65°56′ N & S

(V) is negative between latitudes 45°03′ & 80° N & S

B_5 is negative between latitudes 27°57′ & 76°40′ N & S

A_6 is negative between latitudes 50° & 68° N & S

3. $\gamma = -C$ used in US Army tables

Grid bearing = Azimuth + γ = Azimuth − C

NORTHERN HEMISPHERE λ_0			SOUTHERN HEMISPHERE λ_0	
−	+	E′	−	+
+	+	N′	−	−
+	−	γ	−	+

Computed

Checked

Date Tues 11 Feb '69

5.10 GRID TO SPHEROID, USING US ARMY TABLES

5.10.1 This computation uses Volume 2 of the US Army Tables, TM 5-241-32/2. Given the zone, easting and northing of a point, the latitude, longitude and grid convergence can be calculated. The formulae used in the tables are not quite as accurate as Redfearn's, but suffice for all practical purposes. The tables are designed to give a precision of 0".001 in latitude and longitude, and 0".01 in grid convergence, within 3½° of a central meridian.

5.10.2 See the computation form and example at *5.11.* The central meridian can be determined at once from the table on the form. Convert to true coordinates by the formulae

$E' = E - \quad 500\ 000$ metres
$N' = N - 10\ 000\ 000$ metres in the southern hemisphere
$N' = N$ in the northern hemisphere

5.10.3 The sign convention is unusual. Put $q = E'.10^{-6}$. Then q and the computed latitude ϕ' are always taken positive.

5.10.4 For the latitude ϕ:

$$\phi = \phi' - (\text{VII})\, q^2 + (\text{VIII})\, q^4 - D_6$$

To obtain ϕ', do a reverse interpolation in table (I), by the method explained in paragraph *5.3.2.* For other terms, interpolate in the tables as in paragraph *5.3.1.* The tabular values and increments need not then be recorded. All the tabulated functions are always positive, so that the terms alternate in sign, +, –, +, –. In the southern hemisphere, the lattiude so computed should be preceded by a negative sign.

5.10.5 Similarly, for longitude,

$$\omega = (\text{IX})\, q - (\text{X})\, q^3 + E_5$$

Here the first and third terms are always positive, and the second term always negative. For precise work, (IX) must include the small term Δ^2 (IX). Note that Δ^2 (IX) is always negative, though on many pages of the tables the negative sign has been erroneously omitted. West of a central meridian (i.e., when E′ is negative) ω is also taken negative, and

$$\lambda = \lambda_0 + \omega$$

5.10.6 For grid convergence

$$\gamma = (\text{XV})\, q - (\text{XVI})\, q^3 + F_5$$

The first and third terms are always positive, the second always negative. The sign of γ is taken from the diagram on the form. The sign of γ used in Australia is the opposite of C used in the US Army Tables.

5.11

TRANSFORMATION OF COORDINATES FROM GRID TO GEOGRAPHIC

AUSTRALIAN MAP GRID

ZONE ________ STATION ______ ______ ______

Zone	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Central Meridian	87°	93°	99°	105°	111°	117°	123°	129°	135°	141°	147°	153°	159°	165°	171°	177°E

1	Easting E	
2	False Origin	− 500 000 ·000
3	E′ $\pm$	
4	Northing N	
5	Southern Hemisphere	−10 000 000 ·000
6	N′ $\pm$	
7	(I) Tabular value	
8	6 − 7	
9	ϕ' for N′	
10	(VII) Tabular value	
11	Increment	
12	(VII) for ϕ'	
13	(VIII) for ϕ'	
14	(IX) Tabular value	
15	Increment	
16	Δ^2(IX)	
17	(IX) for ϕ'	
18	(X) Tabular value	
19	Increment	
20	(X) for ϕ'	
21	(XV) Tabular value	
22	Increment	
23	(XV) for ϕ'	
24	(XVI) for ϕ	

25	$q = E'.10^{-6}$	+
26	q^2	+
27	q^3	+
28	q^4	+
29	9	+
30	12.26	−
31	13.28	+
32	D_6 for q	−
33	$29+30+31+32=\phi \pm$ Latitude	
34	17.25	+
35	20.27	−
36	E_5 for q	+
37	$34+35+36 = \omega''$	
38	ω $\pm$	
39	Central Meridian λ_o	
40	$38+39 = \lambda$ Longitude	
41	23.25	+
42	24.27	−
43	F_5 for q	+
44	$41+42+43 = \gamma''$	
45	Grid Convergence γ $\pm$	

$q = E'.\,10^{-6}$

$\phi = \phi' - VII.q^2 + VIII.q^4 - D_6$

$\omega = IX.q - X.q^3 + E_5$

$\lambda = \lambda_o + \omega$

$\gamma = XV.q - XVI.q^3 + F_5$

Computed ______ ______ ______ ______

Checked ______ ______ ______ ______

Date ______ ______ ______ ______

SIGN CONVENTION

NOTE:
1. q & ϕ' are always taken positive
2. Δ^2(IX) is always negative
3. $\gamma = -C$ used in US Army tables

Grid Bearing = Azimuth + γ = Azimuth − C

NORTHERN HEMISPHERE λ_o			SOUTHERN HEMISPHERE λ_o	
−	+	E′	−	+
+	+	N′	−	−
−	+	ω	−	+
+	−	γ	−	+

TRANSFORMATION OF COORDINATES FROM GRID TO GEOGRAPHIC

AUSTRALIAN MAP GRID

ZONE 55

STATION BUNINYONG

Zone	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Central Meridian	87°	93°	99°	105°	111°	117°	123°	129°	135°	141°	147°	153°	159°	165°	171°	177°E

1	Easting E		228 742 ·077
2	False Origin	−	500 000 ·000
3	E′ ±		− 271 257 ·923
4	Northing N		5 828 074 ·208
5	Southern Hemisphere	−	10 000 000 ·000
6	N′ ±		− 4 171 925 ·792
7	(I) Tabular value		4 170 695 ·637
8	6 − 7		1 230 ·155
9	ϕ' for N′		37 41 39 ·9161
10	(VII) Tabular value		1 963 ·129
11	Increment	+	0 ·780
12	(VII) for ϕ'		1 963 ·909
13	(VIII) for ϕ'		27 ·201
14	(IX) Tabular value		40 828 ·438
15	Increment	+	6 ·080
16	Δ^2 (IX)	−	0 ·001
17	(IX) for ϕ'		40 834 ·517
18	(X) Tabular value		366 ·951
19	Increment	+	0 ·214
20	(X) for ϕ'		367 ·165
21	(XV) Tabular value		24 958 ·30
22	Increment	+	9 ·97
23	(XV) for ϕ'		24 968 ·27
24	(XVI) for ϕ		325 ·3

25	$q = E'.\ 10^{-6}$	+	0·271 257923
26	q^2	+	0·073 58086
27	q^3	+	0·019 9594
28	q^4	+	0·005 4141
29	9	+	37 41 39·9161
30	12 . 26	−	02 24·5061
31	13.28	+	0·1473
32	D_6 for q	−	0·0002
33	$29+30+31+32=\phi$ ± Latitude		− 37° 39′ 15″5571
34	17 . 25	+	11 076·6863
35	20 . 27	−	7·3284
36	E_5 for q	+	0·0091
37	$34+35+36 = \omega''$		11 069·3670
38	ω ±		−03 04 29·3670
39	Central Meridian λ_o		147
40	$38+39=\lambda$ Longitude		+143° 55′ 30″6330
41	23 . 25	+	6 772·84
42	24 . 27	−	6·49
43	F_5 for q	+	0·01
44	$41+42+43=\gamma''$		6 766·36
45	Grid Convergence γ ±		−01° 52′ 46″36

$q = E'.\ 10^{-6}$

$\phi = \phi' - \text{VII}.q^2 + \text{VIII}.q^4 - D_6$

$\omega = \text{IX}.q - \text{X}.q^3 + E_5$

$\lambda = \lambda_o + \omega$

$\gamma = \text{XV}.q - \text{XVI}.q^3 + F_5$

Computed M.I SGT

Checked R. [illegible] WO2

Date Tues 11 Feb '69

SIGN CONVENTION

NOTE:
1. $q \,\&\, \phi'$ are always taken positive
2. Δ^2(IX) is always negative
3. $\gamma = -C$ used in US Army tables

Grid Bearing = Azimuth + γ = Azimuth − C

NORTHERN HEMISPHERE			SOUTHERN HEMISPHERE	
λ_o			λ_o	
−	+	E′	−	+
+	+	N′	−	−
−	+	ω	−	+
+	−	γ	−	+

5.12 ZONE TO ZONE TRANSFORMATION

5.12.1 It is sometimes necessary to compute the grid coordinates in a particular zone for a point whose grid coordinates are known in the adjacent zone.

5.12.2 One method is to convert the known coordinates into latitude and longitude using form *5.11,* and then convert back to grid coordinates in the adjacent zone using form *5.9.*

5.12.3 Where geographical coordinates are not required the following direct method may be used. The formulae are adapted from Jordan-Eggert: *Handbuch der Vermessungskunde* (Volume III, second half - volume, section 38, 1941 edition) and are for zone to zone transformations on the Transverse Mercator projection. A computer program is available for this method. Lauf's method (paragraph *1.8.3*) can also be used.

5.12.4 **Formulae**

$E_2 = 500\,000 - E'_z + (E'_1 - E'_z) \cos 2\gamma_z - (N_1 - N_z) \sin 2\gamma_z + H_1 L^2 \sin (2\theta_z + J_1)$

$N_2 = N_z + (N_1 - N_z) \cos 2\gamma_z + (E'_1 - E'_z) \sin 2\gamma_z + H_1 L^2 \cos (2\theta_z + J_1)$

where:

Z = a point on the zone boundary

E_1, N_1 = known coordinates of the point to be transformed

E_2, N_2 = required coordinates of the point in the adjacent zone

θ_z = plane bearing from Z to the point to be transformed

$\tan J_1 = [\omega_z^2 \cos^2 \phi_z (1 + 31 \tan^2 \phi_z) - 6 (1 + e'^2 \cos^2 \phi_z)] / [18\omega_z \sin \phi_z (1 + e'^2 \cos^2 \phi_z)]$

$H_1 = - 3\omega_z^2 \sin \phi_z \cos \phi_z / (\rho_z \cos J_1)$

5.12.5 These formulae are accurate to 10 mm anywhere within half a degree of a zone boundary.

5.12.6 Values of E'_z, $\sin 2\gamma_z$, $(1 - \cos 2\gamma_z)\, 10^4$, H_1 and J_1 are tabulated for the Australian Map Grid in Annex E. Enter the N_z column with N_1 and extract the closest listed value of N_z and the other functions without interpolation.

5.12.7 See the computation form and example at *5.13.* Enter the data on lines 15 and 26, and extract the tabulated values from Annex E for lines 1 to 4, 18 and 27. Always enter 18 as positive. Calculate 19 and 28 and enter 5 and 6. Note that the differences at 17, 19 and 28 are arithmetic, the signs of 19 and 28 being determined from the sign convention. Determine the quadrant of θ_z and compute 7 to 14. Calculate 20 to 25 and 29 to 32. The example illustrates the maximum number of decimal places required.

5.13

ZONE TO ZONE TRANSFORMATION

AUSTRALIAN MAP GRID

ZONE ____________(1)   To ZONE ____________(2)   E to W / W to E   STATION ________________________

$$E_2 = 500000 - E'_z + (E'_1 - E'_z)\cos 2\gamma_z - (N_1 - N_z)\sin 2\gamma_z + H_1 L^2 \sin(2\theta_z + J_1)$$

$$N_2 = N_z + (N_1 - N_z)\cos 2\gamma_z + (E'_1 - E'_z)\sin 2\gamma_z + H_1 L^2 \cos(2\theta_z + J_1)$$

1	$\sin 2\gamma_z$		
2	$(1-\cos 2\gamma_z).10^{-4}$		
3	$H_1.10^8$		
4	J_1	±	
5	19.10^{-4}		
6	28.10^{-4}		
7	5/6 = $\tan\theta_z$		
8	θ_z		
9	$2\theta_z$		
10	4+9 = $(2\theta_z+J_1)$		
11	sin 8		
12	$(5/11)^2$		
13	sin 10		
14	cos 10		

15	E_1		
16	False Origin		500 000.000
17	Difference 15 and 16 = E'_1		
18	E'_z		+
19	Difference 17 and 18	±	
20	1.28	±	
21	2.5	±	
22	3.12.13	±	
23	Sum 18 to 22 = E'_2	±	
24	False Origin		500 000.000
25	23 + 24 = **EASTING** E_2		
26	N_1		
27	N_z		
28	Difference 26 and 27	±	
29	1.19	±	
30	2.6	±	
31	3.12.14	±	
32	Sum 27 to 31 = **NORTHING** N_2		

SIGN CONVENTION

SOUTHERN HEMISPHERE

W Zone to E Zone			E Zone to W Zone	
4 is +		4	4 is −	
23 is −		23	23 is +	
$\theta_z = 0°$ $(2\theta_z+J_1)$			$\theta_z = 0°$ $(2\theta_z+J_1)$	
+	−	19	−	+
+	+	20	+	+
−	+	21	+	−
+	−	(22)	−	+
+	+	28	+	+
−	+	29	+	−
−	−	30	−	−
+	+	(31)	+	+
270° — Z	— 90°		270° — Z	— 90°
+	−	19	−	+
−	−	20	−	−
−	+	21	+	−
+	−	(22)	−	+
−	−	28	−	−
−	+	29	+	−
+	+	30	+	+
−	−	(31)	−	−
$\theta_z = 180°$ $(2\theta_z+J_1)$			$\theta_z = 180°$ $(2\theta_z+J_1)$	

NOTE: 1 Determine the quadrant of θ_z by inspection
2 Determine sign of 22 and 31 using $(2\theta_z+J_1)$

Computed ________________________

Checked ________________________

Date ________________________

ZONE TO ZONE TRANSFORMATION

AUSTRALIAN MAP GRID

ZONE 55 (1) To ZONE 54 (2) E to W / W to E STATION BUNINYONG

$$E_2 = 500000 - E'_z + (E'_1 - E'_z)\cos 2\gamma_z - (N_1 - N_z)\sin 2\gamma_z + H_1 L^2 \sin(2\theta_z + J_1)$$

$$N_2 = N_z + (N_1 - N_z)\cos 2\gamma_z + (E'_1 - E'_z)\sin 2\gamma_z + H_1 L^2 \cos(2\theta_z + J_1)$$

1	$\sin 2\gamma_z$		0·064 1044
2	$(1-\cos 2\gamma_z).10^4$		20568
3	$H_1.10^8$		0·6504
4	J_1	±	- 84° 29′
5	19.10^{-4}		0·6942
6	28.10^{-4}		1·0405
7	$5/6 = \tan\theta_z$		0·667 18
8	θ_z		326° 17′
9	$2\theta_z$		292 34
10	$4+9 = (2\theta_z + J_1)$		208 05
11	$\sin 8$		0·5551
12	$(5/11)^2$		1·5640
13	$\sin 10$		0·4708
14	$\cos 10$		0·8823

15	E_1		228 742·077
16	False Origin		500 000.000
17	Difference 15 and 16 $= E'_1$		271 257·923
18	E'_z		+ 264 315·106
19	Difference 17 and 18	±	- 6 942·817
20	1.28	±	+ 667·000
21	2.5	±	+ 14·278
22	3.12.13	±	- 0·479
23	Sum 18 to 22 $= E'_2$	±	+258 053·088
24	False Origin		500 000.000
25	23 + 24 = **EASTING** E_2		758 053·088
26	N_1		5 828 074·208
27	N_z		5 817 669·298
28	Difference 26 and 27	±	+ 10 404·910
29	1.19	±	+ 445·065
30	2.6	±	- 21·401
31	3.12.14	±	- 0·897
32	Sum 27 to 31 = **NORTHING** N_2		5 828 496·974

SIGN CONVENTION

SOUTHERN HEMISPHERE

W Zone to E Zone			E Zone to W Zone	
4 is +		4	4 is −	
23 is −		23	23 is +	
$\theta_z = 0°$ $(2\theta_z+J_1)$			$\theta_z = 0°$ $(2\theta_z+J_1)$	
+	−	19	−	+
+	+	20	+	+
−	+	21	+	−
+	−	(22)	−	+
+	+	28	+	+
−	+	29	+	−
−	−	30	−	−
+	+	(31)	+	+
270°	Z 90°		270°	Z 90°
+	−	19	−	+
−	−	20	−	−
−	+	21	+	−
+	−	(22)	−	+
−	−	28	−	−
−	+	29	+	−
+	+	30	+	+
−	−	(31)	−	−
$\theta_z = 180°$ $(2\theta_z+J_1)$			$\theta_z = 180°$ $(2\theta_z+J_1)$	

θ

$(2\theta+J_1)$

NOTE: 1 Determine the quadrant of θ_z by inspection
2 Determine sign of 22 and 31 using $(2\theta_z+J_1)$

Computed H Taylor.

Checked

Date 21 Oct 69

5.14 AMG COORDINATES FROM GRID BEARING AND SPHEROIDAL DISTANCE

5.14.1 AMG coordinates and reverse grid bearing can be calculated from grid bearing and spheroidal distance from a known point. The method provides a useful check of the values obtained by the direct problem on the spheroid (paragraphs *5.4* and *5.5*) and the subsequent transformation of geographic coordinates to grid coordinates (paragraphs *5.8* and *5.9*). It may also be used when the geographic coordinates of the new point are not required.

5.14.2 Alternative computations using plane bearing and plane distance, which can give more accurate results, are described in paragraphs *5.16* and *5.20.*

5.14.3 **Formulae**

$$E_2 = 500\,000 + E'_1 + k_0\ s \sin\beta_1 - E'_1\ (k_0\ s\cos\beta_1)^2/2r^2 - k_0 s \sin\beta_1\ (k_0 s \cos\beta_1)^2/6r^2 + (E'^3_2 - E'^3_1)/6r^2$$

$$N_2 = N_1 + k_0\ s\cos\beta_1 + E'^2_2\ k_0\ s\cos\beta_1/2r^2 - k_0\ s\cos\beta_1\ (k_0\ s\sin\beta_1)^2/6r^2$$

$$\beta_2 = \beta_1 \pm 180^o - k_0\ s\cos B_1\ (E'_1 + E'_2)/2r^2\ \sin 1''$$

5.14.4 The last term of the formulae for β_2, including the negative sign, is the Line Curvature $\Delta\beta$.

5.14.5 The formulae are sufficiently accurate for first order lines up to 50 kilometres long anywhere in a zone.

5.14.6 Values of the functions $10^{15}/2r^2$, $10^{15}/6r^2$ and $10^{10}/2r^2\ \sin 1''$ are tabulated in Annex F. The table is best entered with the mean latitude ϕ_m; but it is usually sufficient to enter the table with the northing N of the given station.

5.14.7 See the computation form and example at *5.15.* Enter the data for station 1 in lines 1, 2, 14 and 26. Interpolate in Annex F using ϕ_m or N as argument and enter values in lines 6 to 8. Determine 3 to 5 and 16, entering the correct sign for 16 as it is used to determine the sign of other values. Calculate 17 and 27 and enter 9 to 11. Calculate 18 to 20. Enter 12 and calculate 21 to 25. Enter 13 and calculate 27 to 34. All signs are taken from the sign convention on the form except lines 16, 20 and 23 which are determined by algebraic summation.

GRID COORDINATES FROM GRID BEARING AND SPHEROIDAL DISTANCE

AUSTRALIAN MAP GRID

ZONE ______ From STATION 1 ____________________ To STATION 2 ____________________

$$E_2 = 500\,000 + E_1' + k_0 . s . \sin\beta_1 - E_1' (k_0 . s . \cos\beta_1)^2/2r^2 - k_0 . s . \sin\beta_1 (k_0 . s . \cos\beta_1)^2/6r^2 + (E_2'^3 - E_1'^3)/6r^2$$

$$N_2 = N_1 + k_0 . s . \cos\beta_1 + E_2'^2 . k_0 . s . \cos\beta_1 /2r^2 - k_0 . s . \cos\beta_1 (k_0 . s . \sin\beta_1)^2/6r^2$$

$$\beta_2 = \beta_1 \pm 180^\circ - k_0 s \cos\beta_1 (E_1' + E_2')/2r^2 \sin 1''$$

1	β_1	
2	s	
3	s . 0·9996	
4	$\sin\beta_1$	
5	$\cos\beta_1$	
6	$10^{15}/2r^2$ for N	
7	$10^{15}/6r^2$ for N	
8	$10^{10}/2r^2 \sin 1''$ for N	
9	16.10^{-5}	
10	17.10^{-5}	
11	27.10^{-5}	
12	20.10^{-5}	
13	23.10^{-5}	

14	E_1		
15	False Origin		500 000.000
16	$14 - 15 = E_1'$		
17	3 . 4	±	
18	$11^2 . 9 . 6$	±	
19	$11^2 . 10 . 7$	±	
20	$16 + 17 + 18 + 19 = E_2'$		
21	$12^3 . 7$	±	
22	$9^3 . 7$	±	
23	$20 + 21 + 22 = E_2'$		
24	False Origin		500 000.000
25	23+24 = **EASTING** E_2		
26	N_1		
27	3 . 5	±	
28	$13^2 . 11 . 6$	±	
29	$10^2 . 11 . 7$	±	
30	26+27+28+29 = **NORTHING** N_2		
31	$\beta_1 \pm 180^\circ$		
32	11 . 8 . 9	±	
33	11 . 8 . 13	±	
34	31+32+33 = **BEARING** β_2		

SIGN CONVENTION

18 & 22 are opposite 16
21 is the same as 20

$\beta_1 = 0^\circ$

−	17	+
+	19	−
+	27	+
+	28	+
−	29	−
opposite 16	32	opposite 16
opposite 23	33	opposite 23

270° — 1 — 90°

−	17	+
+	19	−
−	27	−
−	28	−
+	29	+
same as 16	32	same as 16
same as 23	33	same as 23

$\beta_1 = 180^\circ$

Computed ____________________

Checked ____________________

Date ____________________

GRID COORDINATES FROM GRID BEARING AND SPHEROIDAL DISTANCE

AUSTRALIAN MAP GRID

ZONE 54 From STATION 1 BUNINYONG To STATION 2 FLINDERS PEAK

$$E_2 = 500\,000 + E_1' + k_0.s.\sin\beta_1 - E_1'(k_0.s.\cos\beta_1)^2/2r^2 - k_0.s.\sin\beta_1(k_0.s.\cos\beta_1)^2/6r^2 + (E_2'^3 - E_1'^3)/6r^2$$

$$N_2 = N_1 + k_0.s.\cos\beta_1 + E_2'^2.k_0.s.\cos\beta_1/2r^2 - k_0.s.\cos\beta_1(k_0.s.\sin\beta_1)^2/6r^2$$

$$\beta_2 = \beta_1 \pm 180^\circ - k_0 s\cos\beta_1(E_1'+E_2')/2r^2\sin 1''$$

1	β_1	128° 57′ 43″·75
2	s	54 972 ·161
3	s . 0·9996	54 950 ·172
4	sin β_1	0 · 777 561 49
5	cos β_1	0 · 628 806 91
6	$10^{15}/2r^2$ for N	12 · 321 64
7	$10^{15}/6r^2$ for N	4 · 107 22
8	$10^{10}/2r^2 \sin 1''$ for N	25 · 415 21
9	16.10^{-5}	2 · 580 530 90
10	17.10^{-5}	0 · 427 271 38
11	27.10^{-5}	0 · 345 530 48
12	20.10^{-5}	3 · 007 762 22
13	23.10^{-5}	3 · 008 174 01

14	E_1		758 053·090
15	False Origin		500 000.000
16	$14-15 = E_1'$		+ 258 053·090
17	3 . 4	±	+ 42 727·138
18	$11^2.9.6$	±	− 3 · 796
19	$11^2.10.7$	±	− 0 · 210
20	$16+17+18+19 = E_2'$		+ 300 776 · 222
21	$12^3.7$	±	+ 111 · 758
22	$9^3.7$	±	− 70 · 579
23	$20+21+22 = E_2'$		+ 300 817 · 401
24	False Origin		500 000.000
25	23+24 = **EASTING E_2**		800 817 ·401
26	N_1		5 828 496 · 974
27	3.5	±	− 34 553 · 048
28	$13^2.11.6$	±	− 38 · 527
29	$10^2.11.7$	±	+ 0 · 259
30	26+27+28+29 = **NORTHING N_2**		5 793 905 · 658
31	$\beta_1 \pm 180^\circ$		308° 57′ 43″·75
32	11 . 8 . 9	±	+ 22 ·66
33	11 . 8 . 13	±	+ 26 ·42
34	31+32+33 = **BEARING β_2**		308° 58′ 32″·83

SIGN CONVENTION

18 & 22 are opposite 16
21 is the same as 20

$\beta_1 = 0^\circ$

−	17	+
+	19	−
+	27	+
+	28	+
−	29	−
opposite 16	32	opposite 16
opposite 23	33	opposite 23

270° — 1 — 90°

−	17	+
+	19	−
−	27	−
−	28	−
+	29	+
same as 16	32	same as 16
same as 23	33	same as 23

2

$\beta_1 = 180^\circ$

Computed

Checked

Date 15 Oct 69

5.16 AMG COORDINATES FROM PLANE BEARING AND PLANE DISTANCE

5.16.1 In this computation the plane bearing θ is obtained by adding algebraically the arc-to-chord correction δ to the grid bearing β. The plane distance L, which for practical purposes is equal to the grid distance S, is obtained by multiplying the spheroidal distances by the line scale factor K. Grid coordinates are then computed from plane bearing and plane distance using plane trigonometry.

5.16.2 **Formulae**

$$\delta_1'' = -(N_2 - N_1)(E_2' + 2E_1')\ \underline{[1 - (E_2' + 2E_1')^2/27r^2]}/6r^2 \sin 1''$$

$$\delta_2'' = (N_2 - N_1)(2E_2' + E_1')\ \underline{[1 - (2E_2' + E_1')^2/27r^2]}/6r^2 \sin 1''$$

$$\beta_1 = \theta_1 - \delta_1 \qquad \beta_2 = \theta_1 \pm 180^\circ - \delta_2$$

$$K = k_0 \{1 + [(E_1'^2 + E_1' E_2' + E_2'^2)/6r_2]\ \underline{[1 + (E_1'^2 + E_1' E_2' + E_2'^2)/36r^2]}\}$$

$$L = sK$$

where $E_2' \approx E_1' + k_1\ s \sin\beta_1$

$N_2 - N_1 \approx k_1\ s \cos\beta_1$

5.16.3 These formulae are accurate to 0''02 and 0·1 ppm over any 100 kilometre line in an AMG zone.

5.16.4 The computation form and example in paragraph *5.17* omit the terms underlined as for most practical purposes their effects are negligible. For a line 100 kilometres long running north and south on a zone boundary, the errors are 0''08 and 0·25 ppm, about 25 mm respectively.

5.16.5 The computation of traverses, using a version of these formulae, is described in paragraph *5.20.*

5.16.6 See the computation form and example at *5.17.* Enter the data on lines 1, 19, 25 and 32. Determine 27 and 9, entering the correct sign as it is used to determine the sign of other values. Enter 2 to 4 by interpolation in Annex G using E'_1 as argument, and in Annex F using ϕ_m or N. Complete 5 to 24 determining L and θ. Calculate 28 to 38. All signs are taken from the sign convention on the form except lines 27, 9, 11 and 29 which are determined by algebraic summation.

5.17

GRID COORDINATES FROM PLANE BEARING AND PLANE DISTANCE

AUSTRALIAN MAP GRID

ZONE ________ **From STATION 1** ______________________ **To STATION 2** ______________________

$$\theta = \beta_1 - [(N_2 - N_1)(2E_1' + E_2')/6r^2 \sin 1'']$$

$$L = s\,k_0[1 + (E_1'^2 + E_1' . E_2' + E_2'^2)/6r^2]$$

$$E_2 = 500\,000 + E_1' + L \sin\theta$$

$$N_2 = N_1 + L \cos\theta$$

$$\beta_2 = \theta \pm 180° - [(N_2 - N_1)(2E_2' + E_1')/6r^2 \sin 1'']$$

where $E_2' \approx E_1' + k_1 . s . \sin\beta_1$

$N_2 - N_1 \approx k_1 . s . \cos\beta_1$

1	s		
2	k_1 for E_1'		
3	$10^{15}/6r^2$ for N		
4	$10^{111}/6r^2 \sin 1''$ for N		
5	$2.1.10^{-5}$		
6	sin 19		
7	cos 19		
8	4.5.7		
9	27.10^{-5}		
10	5.6	±	
11	9+10		
12	9^2		
13	9.11		
14	11^2		
15	12+13+14		
16	1.0·9996		
17	$3.15.16.10^{-5}$		
18	16+17=L		

19	β_1		
20	2.9.8	±	
21	11.8	±	
22	19+20+21=θ		
23	sin 22		
24	cos 22		
25	E_1		
26	False Origin		500 000.000
27	25−26=E_1'		
28	18.23	±	
29	27+28=E_2'		
30	False Origin		500 000.000
31	29+30= **EASTING** E_2		
32	N_1		
33	18.24	±	
34	32+33= **NORTHING** N_2		
35	22±180°		
36	$2.29.10^{-5}.8$	±	
37	20/2	±	
38	35+36+37= **BEARING** β_2		

SIGN CONVENTION

(β_1=0°)
θ=0°

− (10) +
opposite 9 (20) opposite 9
opposite 11 (21) opposite 11
− 28 +
+ 33 +
opposite 29 36 opposite 29
opposite 27 37 opposite 27

270° ———— 1 ———— 90°

− (10) +
same as 9 (20) same as 9
same as 11 (21) same as 11
− 28 +
− 33 −
same as 29 36 same as 29
same as 27 37 same as 27

(β_1=180°)
θ=180°

Computed ______________________

Checked ______________________

Date ______________________

GRID COORDINATES FROM PLANE BEARING AND PLANE DISTANCE

AUSTRALIAN MAP GRID

ZONE 54 **From STATION 1** BUNINYONG **To STATION 2** FLINDERS PEAK

$$\theta = \beta_1 - [(N_2 - N_1)(2E_1' + E_2')/6r^2 \sin 1'']$$
$$L = s k_0 [1 + (E_1'^2 + E_1' . E_2' + E_2'^2)/6r^2]$$
$$E_2 = 500\,000 + E_1' + L \sin\theta$$
$$N_2 = N_1 + L\cos\theta$$
$$\beta_2 = \theta \pm 180^\circ - [(N_2 - N_1)(2E_2' + E_1')/6r^2 \sin 1'']$$

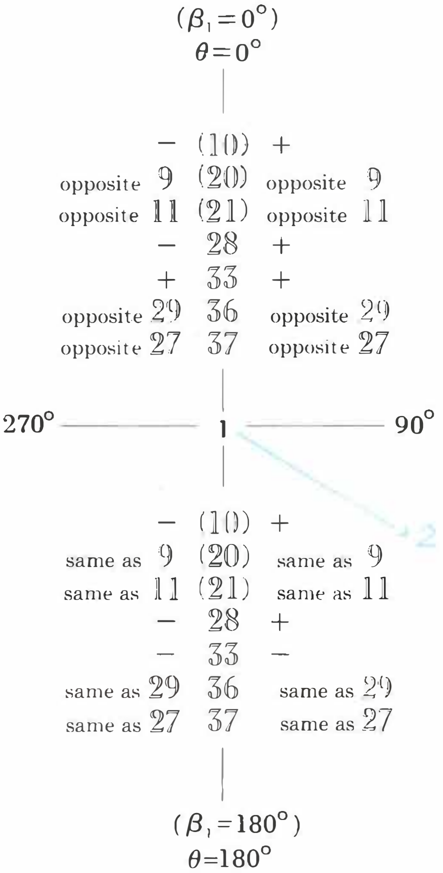
where $E_2' \approx E_1' + k_1 . s . \sin\beta_1$

$N_2 - N_1 \approx k_1 . s . \cos\beta_1$

1	s		54 972 · 161
2	k_1 for E_1'		1 · 000 4
3	$10^{15}/6r^2$ for N		4 · 107 2
4	$10^{10}/6r^2 \sin 1''$ for N		8 · 471 7
5	$2.1.10^{-5}$		0 · 549 72
6	sin 19		0 · 777 56
7	cos 19		0 · 628 81
8	4.5.7		2 · 928 40
9	27.10^{-5}		+ 2 · 580 53
10	5.6	±	+ 0 · 427 44
11	9 + 10		3 · 007 97
12	9^2		6 · 659 14
13	9.11		7 · 762 16
14	11^2		9 · 047 88
15	12+13+14		23 · 469 18
16	1 . 0·9996		54 950 · 172
17	$3.15.16.10^{-5}$		52 · 968
18	16+17 = L		55 003 · 140

19	β_1		128 57 43 · 75
20	2.9.8	±	+ 15 · 11
21	11.8	±	+ 8 · 81
22	19 + 20 + 21 = θ		128 58 07 · 67
23	sin 22		0 · 777 488 57
24	cos 22		0 · 628 897 06
25	E_1		758 053 · 090
26	False Origin		500 000 . 00
27	25 − 26 = E_1'		+ 258 053 · 090
28	18.23	±	+ 42 764 · 313
29	27 + 28 = E_2'		+ 300 817 · 403
30	False Origin		500 000 . 00
31	29 + 30 = **EASTING** E_2		800 817 · 403
32	N_1		5 828 496 · 974
33	18.24	±	− 34 591 · 313
34	32 + 33 = **NORTHING** N_2		5 793 905 · 661
35	22 ± 180°		308 58 07 · 67
36	$2.29.10^{-5}.8$	±	+ 17 · 62
37	20/2	±	+ 7 · 56
38	35 + 36 + 37 = **BEARING** β_2		308° 58′ 32″ 85

SIGN CONVENTION



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5.18 GRID BEARINGS AND SPHEROIDAL DISTANCE FROM AMG COORDINATES

5.18.1 The following formulae provide the only direct method for obtaining grid bearings and spheroidal distance from AMG coordinates:

$$\begin{aligned}
\tan\theta_1 &= (E'_2 - E'_1)/(N_2 - N_1) \text{ OR } \cot\theta_1 = (N_2 - N_1)/(E'_2 - E'_1) \\
L &= (E'_2 - E'_1)/\sin\theta_1 = (N_2 - N_1)/\cos\theta_1 \\
K &= k_0 \left\{ 1 + [(E_1'^2 + E'_1 + E_2'^2)/6r^2] \; \underline{[1 + (E_1'^2 + E'_1 E'_2 + E_2'^2)/36r^2]} \right\} \\
s &= L/K \\
\delta''_1 &= -(N_2 - N_1)(E'_2 + 2E'_1) \; \underline{[1 - (E'_2 + 2E'_1)^2/27r^2]} /6r^2 \sin 1'' \\
\delta''_2 &= (N_2 - N_1)(2E'_2 + E'_1) \; \underline{[1 - (2E'_2 + E'_1)^2/27r^2]/6r^2} \sin 1'' \\
\beta_1 &= \theta_1 - \delta_1 \qquad \beta_2 = \theta_1 \pm 180^\circ - \delta_2
\end{aligned}$$

5.18.2 The computation form and numerical example at *5.19* omit the terms underlined as for most practical purposes their effects are negligible. On a line 100 kilometres long running north and south on a zone boundary the errors are 0$''$08 and 0·25 ppm, i.e. about 25 mm.

5.18.3 See the computation form and example at *5.19.* Enter the data on lines 1, 4, 7 and 8. Interpolate in Annex F using ϕ_m or a northing value as argument and enter values in lines 15 and 16. Compute 3, 6, 9 and 10 including the signs of 3 and 6 as they determine the signs of the functions listed in the sign convention. Divide the smaller by the larger of lines 9 and 10, enter the appropriate line 11 and determine 12. Indicate the quadrant of θ on the sign convention using the coordinates of the two stations. Compute the plane distance L using both sets of lines 13 and 14 as a check, but where small differences occur adopt the value computed from the larger of $\sin\theta_1$ or $\cos\theta_1$. Enter 17 to 19, compute 21 to 24 and enter 20. Compute 25 to 35.

5.19

GRID BEARINGS AND SPHEROIDAL DISTANCE FROM GRID COORDINATES

AUSTRALIAN MAP GRID

ZONE ________ **From STATION 1** ____________________ **To STATION 2** ____________________

$$\tan\theta_1 = (E_2' - E_1')/(N_2 - N_1) \quad \text{OR} \quad \cot\theta_1 = (N_2 - N_1)/(E_2' - E_1')$$

$$L = (E_2' - E_1')/\sin\theta_1 = (N_2 - N_1)/\cos\theta_1$$

$$S = L/k_0\,[1 + (E_1'^2 + E_1'.E_2' + E_2'^2)/6r^2]$$

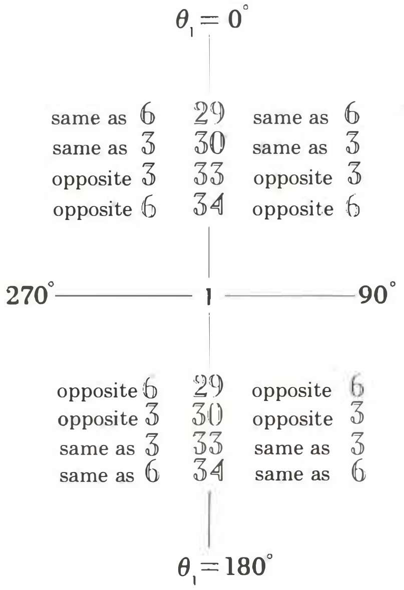
$$\beta_1 = \theta_1 + (N_2 - N_1)(E_2' + 2E_1')/6r^2 \sin 1''$$

$$\beta_2 = \theta_1 \pm 180^\circ - (N_2 - N_1)(2E_2' + E_1')/6r^2 \sin 1''$$

1	E_2	
2	False Origin	500 000.000
3	$1-2=E_2'$	
4	E_1	
5	False Origin	500 000.000
6	$4-5=E_1'$	
7	N_2	
8	N_1	
9	$7-8$	
10	$3-6$	
11	$10/9=\tan\theta_1$	
	OR (select the smaller)	
11	$9/10=\cot\theta_1$	
12	θ_1	
13	$\sin\theta_1$	
14	$10/13=L$	
	AND adopt 14 from larger 13	
13	$\cos\theta_1$	
14	$9/13=L$	
15	$10^{15}/6r^2$ for N	
16	$10^{10}/6r^2 \sin 1''$ for N	
17	3.10^{-5}	
18	6.10^{-5}	
19	9.10^{-5}	
20	24.10^{-5}	

21	17^2		
22	18^2		
23	17.18		
24	21+22+23		
25	(15.20)+1		
26	25.0.9996=K		
27	14/26 = Distance s		
28	12		
29	2.18.19.16	±	
30	33/2	±	
31	28+29+30 = Bearing β_1		
32	12 ± 180°		
33	2.17.19.16	±	
34	29/2	+	
35	32+33+34 = Bearing β_2		

SIGN CONVENTION



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Date ____________________

GRID BEARINGS AND SPHEROIDAL DISTANCE FROM GRID COORDINATES

AUSTRALIAN MAP GRID

ZONE 54 **From STATION 1** BUNINYONG **To STATION 2** FLINDERS PEAK

$\tan\theta_1 = (E_2' - E_1')/(N_2 - N_1)$ OR $\cot\theta_1 = (N_2 - N_1)/(E_2' - E_1')$

$L = (E_2' - E_1')/\sin\theta_1 = (N_2 - N_1)/\cos\theta_1$

$S = L/k_0[1 + (E_1'^2 + E_1'.E_2' + E_2'^2)/6r^2]$

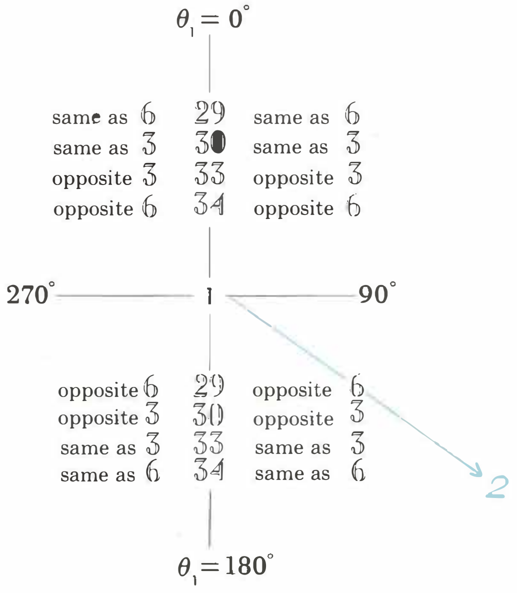
$\beta_1 = \theta_1 + (N_2 - N_1)(E_2' + 2E_1')/6r^2 \sin 1''$

$\beta_2 = \theta_1 \pm 180° - (N_2 - N_1)(2E_2' + E_1')/6r^2 \sin 1''$

1	E_2	800 817 ·407
2	False Origin	500 000 . 000
3	$1-2=E_2'$	300 817 ·407
4	E_1	758 053 · 090
5	False Origin	500 000 . 000
6	$4-5=E_1'$	+ 258 053 · 090
7	N_2	5 793 905 ·650
8	N_1	5 828 496 · 974
9	$7-8$	34 591 ·324
10	$3-6$	42 764 ·317
11	$10/9 = \tan\theta_1$	
	OR (select the smaller)	
11	$9/10 = \cot\theta_1$	0·808 882 88
12	θ_1	128 58 07·69
13	$\sin\theta_1$	0·777 488 51
14	$10/13 = L$	55 003 ·150
	AND adopt 14 from larger 13	
13	$\cos\theta_1$	0·628 897 15
14	$9/13 = L$	55 003 ·150
15	$10^{15}/6r^2$ for N	4·107 04
16	$10^{10}/6r^2 \sin 1''$ for N	8·471 38
17	3.10^{-5}	3·008 17
18	6.10^{-5}	2·580 53
19	9.10^{-5}	0·345 91
20	24.10^{-5}	0·000 234 71

21	17^2		9·049 08
22	18^2		6·659 13
23	17.18		7·762 67
24	21 + 22 + 23		23·470 88
25	(15.20) + 1		1·000 963 96
26	25 . 0.9996 = K		1·000 563 57
27	14/26 = **Distance** s		54 972 ·169
28	12		128 58 07·69
29	2.18.19.16	±	−15 ·12
30	33/2	±	− 8 ·81
31	28 + 29 + 30 = **Bearing** β_1		128 57 43 · 76
32	12 ± 180°		308 58 07·69
33	2.17.19.16	±	+17·62
34	29/2	+	+ 7·56
35	32 + 33 + 34 = **Bearing** β_2		308 58 32·87

SIGN CONVENTION



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Date 15 Oct 69

5.20 TRAVERSE COMPUTATION USING ARC-TO-CHORD CORRECTIONS AND LINE SCALE FACTORS

5.20.1 This method can be varied to suit the requirements of the job. At one extreme, the arc-to-chord corrections and line scale factors can be ignored and the traverse computed using the computation form *5.22* and the formulae of plane trigonometry. At the other extreme, the arc-to-chord corrections and line scale factors can be computed precisely, and the method becomes first order anywhere in an AMG zone. The precision obtained can be closely balanced against the labour involved. Arc-to-chord corrections rarely approach 100″ in magnitude, and corrections to lengths rarely attain 50 metres, so that even for the highest precision much of the work can be done with four significant figures. Prior to precise computation, approximate coordinates and bearings may be carried through the traverse, using uncorrected field measurements, to ensure that the observations are free of gross errors. A diagram of the traverse approximately to scale is desirable.

5.20.2 **Basic Outline**

There are many ways of arranging the computation. Essentially, the work is split into three stages:

(1) Approximate eastings and northings are computed from observed angles and distances.
(2) Arc-to-chord corrections and line scale factors are computed from the approximate coordinates and applied to the observations to give plane angles and plane distances.
(3) Precise coordinates are computed by plane trigonometry.

5.20.3 In the method given below, each line is rigorously computed before the next line is calculated, so that errors in the approximate coordinates do not accumulate. True eastings E′ and differences in northing ΔN are the quantities carried through the computation. Short cuts are used; results are accumulated on the machine; sign conventions are disregarded and signs determined by inspection of a diagram; and the decimal point in the formation of the scale factor is fixed by inspection of a table.

5.20.4 **Formulae and Symbols**

Formulae for arc-to-chord corrections and line scale factors are given at *5.16.2*. If the terms in square brackets are omitted, the errors for a 100 kilometre line running north and south on a zone boundary do not exceed 0″08 in bearing and 0·25 ppm in distance. As the final coordinates of a traverse will be computed and adjusted by least squares on an electronic computer, the small terms in square brackets need never be used for computations on a desk machine. For traverses of lower order, the simplified formulae given in paragraphs *6.10* and *6.11* should be used. For short lines near a central meridian it may be possible to omit the arc-to-chord corrections, and line scale factors and compute the traverse with observed angles and distances, using the formulae of plane trigonometry on form *5.22*.

In this section, the symbol δ_{21} is used for the arc-to-chord correction at station 2 to station 1 and δ_{23} for the correction at station 2 to station 3. If the angles are measured clockwise from station 1 to station 3, then the plane angle P_2 at station 2 is obtained from the observed angle 0_2 by:

$$P_2 = 0_2 + \delta_{23} - \delta_{21}$$

If the angles are measured anti-clockwise, the signs preceding the arc-to-chord corrections in this formula must be reversed; the recording of anti-clockwise angles is therefore discouraged. The signs of the arc-to-chord corrections follow automatically from the formulae, but they can be obtained or checked from the traverse diagram. On the projection, the line bows away from the central meridian, except in the rare case of a line which crosses the central meridian less than one-third of its length from one end, when the bow is determined by the longer part.

5.20.5 **Computations of Arc-to-Chord Corrections and Scale Factors**

See the computation form and example at *5.21*. On this form, few intermediate results are recorded, and the maximum use is made of the accumulator on the calculating machine. This saves time and minimises transcription errors; but computers using this method infrequently, may find that they prefer to record more intermediate results than are shown here. In the first column, enter the name of the station to which the first back bearing has been observed. Leave a blank line, then enter station names and observed distances on alternate lines. In the second column, adjacent to the station names, enter the observed angles. Compute the arc-to-chord correction for the initial back bearing from the known coordinates. In the fifth column, enter $E'10^{-5}$ for both stations and on the top line of column six enter $\Delta N\ 10^{-5}$. Multiplication of all coordinates and differences by 10^{-5} is a great aid in keeping track of the decimal point. δ_{10} can now be calculated on the machine without recording any intermediate results and using the simplest formulae which will give the accuracy required. The value of $10^{10}/6r^2 \sin 1''$ is taken from Annex F; four figures suffice, and interpolation is not necessary. Enter δ_{10} in column 7.

Before computing δ_{12} obtain preliminary values of E'_2 and ΔN_{12} as follows: record the known plane bearing to the RO on the second line in column 2, and add the degrees and minutes only of the angle at the opening station to get an approximate bearing to station 2. Set the observed distance on the machine, look up the sine of this bearing to five figures, transfer it directly to the machine, multiply to obtain $\Delta E'$, add E', and record $E'\ 10^{-5}$ for station 2 in column 5. Do likewise with the cosine of the bearing to obtain ΔN and enter $\Delta N\ 10^{-5}$ in column 6.

Now compute δ_{12} and record it in column 8. Record in column 2 the precise plane bearing from station 1 to station 2 by adding the seconds for the following quantities on the machine:

$$P_{12} = P_{10} + 0_1 + \delta_{12} - \delta_{10}$$

The degrees and minutes have already been entered, but the second figure of the minutes will need to be checked. Record the sine and cosine of the precise plane bearing to as many significant figures as are necessary in columns 3 and 4.

Compute the line scale factor and record it in column 9 without recording any intermediate result. If the line crosses the central meridian $(E'_1\ E'_2)$ is negative. The correct decimal place in the scale factor is obtained by shifting the decimal of the value of $(E'^2_1 + E'_1\ E'_2 + E'^2_2)$ five places after multiplying and taking $10^{15}/6r^2$ directly from Annex F. Four significant figures will usually suffice, and interpolation may be necessary only near a zone boundary.

The position of the decimal point can also be obtained or checked by comparison with the previous line, or from the following table, which is printed on the form:

Mean value of $E'.10^{-5}$	K/k_0
·03	1·000 000 1
·10	1·000 001
·30	1·000 01
·90	1·000 1
2·85	1·001

After multiplying by k_0 a further check can be obtained from Annex G.

Multiply the spheroidal distance by the line scale factor to give the plane distance, and record it in column 9. Compute precise values of E' and ΔN and record them in columns 5 and 6.

Repeat for succeeding lines through the traverse.

To compute δ for the closing bearing, obtain and enter $E'10^{-5}$ for the last station and $\Delta N\ 10^{-5}$ between the closing stations. The closure in bearing and in easting is available directly. To obtain the closure in northing, it is necessary to sum on the machine the northing for the first station and all the computed values of ΔN.

If on any particular line there is doubt whether the initial values of E' and ΔN are sufficiently accurate, the computation for that line can be iterated immediately before passing to the next line.

5.20.6 Plane Traverse Computation

The misclosure in bearing can be adjusted if desired. Compute eastings and northings on the plane traverse form *5.22*. Compute all the eastings first, without clearing the machine, and then compute the northings. The misclosure in coordinates can be adjusted if desired. If it is not desired to adjust the misclosure in bearing, forms *5.21* and *5.22* can be used simultaneously, the precise values of L, sin θ and cos θ being recorded on *5.22* only. The method is very flexible, and different computers will prefer different alternatives.

ARC TO CHORD CORRECTIONS AND LINE SCALE FACTORS

AUSTRALIAN MAP GRID

DECIMAL POINT OF SCALE FACTOR

$E'.10^{-5}$	K / k_0
.03	1.000 0001
.10	1.000 001
.30	1.000 01
.90	1.000 1
2.85	1.001

Traverse from ______________________ To ______________________ Zone __________

$\delta''_{12} = -(N_2 - N_1)(2E'_1 + E'_2)/6r^2 \sin 1''$

$K = k_0[1 + (E'^2_1 + E'_1 E'_2 + E'^2_2)/6r^2]$

	Station	Spheroidal angle		sin θ ±	cos θ ±	E'_1 ±		$\approx \Delta N.10^{-5}$ ±				$\delta_{23} - \delta_{21} = \delta_2$		Line Scale Factor K	
	Spheroidal Distance s	Plane bearing θ				$\approx E'_2.10^{-5}$ ±		ΔN ±		δ_{21}		δ_{23}		Plane Distance	
	1	2		3	4	5		6		7		8		9	
0						0		0-1							
	—	1-0				1		0-1		1-0		1-2			
1		1				1		1 2				δ1			
		1-2				2		1-2		2-1		2-3		1-2	
2		2				2		2-3				δ2			
		2-3				3		2-3		3-2		3-4		2-3	
3		3				3		3-4				δ3			
		3-4				4		3-4		4-3		4-5		3-4	
4		4				4		4-5				δ4			
		4-5				5		4-5		5-4		5-6		4-5	
5		5				5		5-6				δ5			
		5-6				6		5-6		6-5		6-7		5-6	
6		6				6		6-7				δ6			
		6-7				7		6-7		7-6		7-8		6-7	
7		7				7		7-8				δ7			
		7-8				8		7-8		8-7		8-9		7-8	
8		8				8		8-9				δ8			
		8-9				9		8-9		9-8		9-10		8-9	
9		9				9		9-10				δ9			
		9-10				10		9-10		10-9		10		9-10	
10		10				10		10				10			

Computed ______________________ Checked ______________________ Date ______________

ARC TO CHORD CORRECTIONS AND LINE SCALE FACTORS

AUSTRALIAN MAP GRID

DECIMAL POINT OF SCALE FACTOR

$E'.10^{-5}$	K / k_0
.03	1.000 0001
.10	1.000 001
.30	1.000 01
.90	1.000 1
2.85	1.001

Traverse from Buninyong To Arthur's Seat Zone 55

$\delta''_{12} = -(N_2 - N_1)(2E'_1 + E'_2)/6r^2 \sin 1''$

$K = k_0[1 + (E'^2_1 + E'_1 E'_2 + E'^2_2)/6r^2]$

	Station		Spheroidal angle	$\sin\theta$ ±	$\cos\theta$ ±		E'_1 ±		$\approx \Delta N.10^{-5}$ ±				$\delta_{23} - \delta_{21} = \delta_2$		Line Scale Factor K
	Spheroidal Distance s		Plane bearing θ				$\approx E'_2.10^{-5}$ ±		ΔN ±		δ_{21}		δ_{23}		Plane Distance
	1		2	3	4		5		6		7		8		9
0	Smeaton					0	-2·6743	0-1	-0·3964						
		1-0	5° 30′ 57″·86			1	-2·7126	0-1		1-0	+27·20	1-2	-20·70		
1	Buninyong	1	119° 47′ 10″·06			1	-271 257·323	1 2	-0·3177			δ1	-47·90		1·000 3646
	54 972·161	1-2	125 17 20·02	+0·816 24958	-0·577 69942	2	-2·2639	1-2	-31 768·964	2-1	+19·49	2-3	-12·23	1-2	54 992·204
2	Flinders Peak	2	196 43 49·44			2	-226 370·560	2-3	-0·2180			δ2	-31·72		1·000 1849
	27 659·183	2-3	142 00 37·74	+0·61551728	-0·783 12339	3	-2·0935	2-3	-21 802·880	3-2	+11·31	3-4	-11·01	2-3	27 664·237
3	Bellarine	3	163 45 32·33			3	-209 342·707	3-4	-0·2173			δ3	-22·92		1·000 0657
	37 175·169	3-4	125 45 47·15	+0·81144041	-0·584 43517	4	-1·7918	3-4	-21 727·903	4-3	+10·45	4-5	-5·48	3-4	37 177·611
4	Arthurs Seat	4	158 34 37·48			4	-179 175·291	4-5	-0·1333			δ4	-15·93		
		4-5	104 20 08·68			5	-1·2701	4-5		5-4		5-6		4-5	
5	Bass	5				5		5-6				δ5			
		5-6				6		5-6		6-5		6-7		5-6	
6	Fixed Bearing	6	104 20 08·76		Calculated Coords	6	320 824·709	6-7	5752 774·461			δ6			
	Misclose	6-7	+0·08		Fixed Coords	7	320 824·691	6-7	5752 774·441	7-6		7-8		6-7	
7		7			Misclose	7	-0·018	7-8	-0·020			δ7			
		7-8				8		7-8		8-7		8-9		7-8	
8		8				8		8-9				δ8			
		8-9				9		8-9		9-8		9-10		8-9	
9		9				9		9-10				δ9			
		9-10				10		9-10		10-9		10		9-10	
10		10				10		10				10			

Computed Checked Date 12 Nov 1969

5.22

PLANE TRAVERSE COMPUTATION

AUSTRALIAN MAP GRID

Traverse from ____________________ To ____________________ Zone ________

$E_2 = E_1 + L.\sin\theta_1$ $N_2 = N_1 + L.\cos\theta_1$

	Station Plane Distance L	Angle Plane Bearing θ	$\sin\theta \pm$	$\cos\theta \pm$	EASTING	NORTHING
	1	2	3	4	5	6
0						
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						

Computed ____________________ Checked ____________________ Date ____________

5.22

PLANE TRAVERSE COMPUTATION

AUSTRALIAN MAP GRID

Traverse from Buninyong To Arthur's Seat Zone 55

$E_2 = E_1 + L.\sin\theta_1$ $N_2 = N_1 + L.\cos\theta_1$

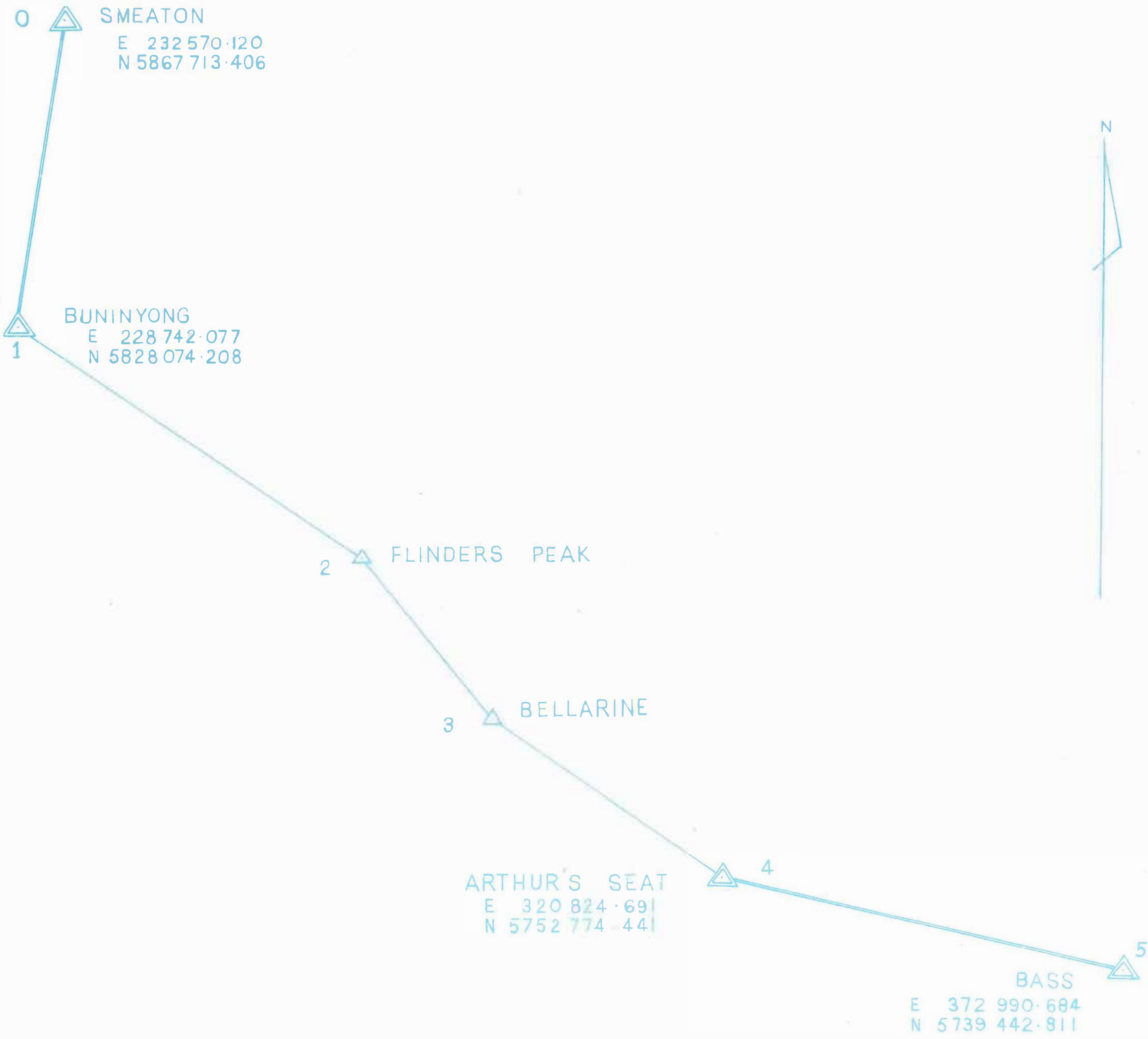
	Station Plane Distance L	Angle Plane Bearing θ	sin θ ±	cos θ ±	EASTING	NORTHING
	1	2	3	4	5	6
0	Smeaton				232 570·120	5867 713·406
		5 30 57·86				
1	Buninyong				228 742·077	5828 074·208
	54 992·204	125 17 20·02	+0·816 24958	-0·577 69942		
2	Flinders Peak				273 629·440	5796 305·244
	27 864·297	142 00 37·74	+0.615 51728	-0·788 12339		
3	Bellarine				290 657·233	5774 502·364
	37 177·611	125 45 47·15	+0·811 44041	-0·584 43517		
4	Arthur's Seat				320 824·709	5752 774·461
		104 20 08·68				
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						

Computed

Checked

Date 12 Nov 69

TRAVERSE: BUNINYONG - ARTHUR'S SEAT
SHOWING FIXED DATA



Approximate Scale: 20km to an inch

6 Simplified Formulae

6.1 AIM

6.1.1 The aim of this chapter is to provide simplified formulae with computation forms and examples for use when a quick check on current work is required, or when third order accuracy suffices.

6.1.2 In general, the accuracy aimed for in this chapter is about 0·3 metres in position and about 1″ in azimuth on lines of up to 40 kilometres. It may be possible to speed up the computation, when lower accuracy is acceptable, by working with fewer figures and by interpolating in tables by inspection. However, when interpolating in tables in this way, it is very easy to make mistakes, and precise interpolation with a desk calculator by the methods given in paragraph *5.3* may be the quickest way of getting the correct answer.

6.2 LATITUDE AND LONGITUDE FROM SPHEROIDAL DISTANCE AND AZIMUTH – SIMPLIFIED PUISSANT'S FORMULAE

6.2.1 The method used is similar to the precise version of Puissant's formulae described in paragraph *5.4;* but the difference between the sine and radian measure of small angles is ignored, the F term is omitted in computing the reverse azimuth, and it will nearly always suffice to drop the last two figures of all tabulated functions. The volume TM 5-241-33 of Latitude Functions of the U.S. Army Tables is required.

6.2.2 **Formulae**

$$\Delta'\phi'' = B_1 \ s \cos a_1 - C_1 \ s^2 \sin^2 a_1 - (B_1 \ s \cos a_1)\, E_1 \ s^2 \sin^2 a_1$$

$$\Delta\phi'' = \Delta'\phi'' - D_1 \ (\Delta'\phi'')^2$$

$$\Delta\lambda'' = A_2 \ s \sin a_1 / \cos \phi_2$$

$$\Delta a'' = \Delta\lambda'' \sin \phi_m \ / \cos (\Delta\phi/2)$$

6.2.3 See the computation form and example at *6.3.* Enter the data on lines 1 to 4 and enter 8 to 11 from TM 5-241-33. Complete 5 to 7 and 12, and compute the latitude. Enter 25 from the tables using ϕ_2 as argument. In many cases it will be sufficient to use ϕ_1 and extract the value when entering 8 to 11. Enter values for 26, 31 and 32. Compute the longitude and reverse azimuth.

6.2.4 On short lines the latitude terms using D and E may be negligible so that lines 10, 11, 15 and 17 may be left blank.

6.3

LATITUDE AND LONGITUDE FROM DISTANCE AND AZIMUTH

SIMPLIFIED PUISSANT'S FORMULAE

From STATION 1 ____________________ To STATION 2 ____________________

1	ϕ_1			
2	λ_1			
3	α_1			
4	s (metres)			
5	$s . 10^{-4}$			
6	$\sin\alpha_1$			
7	$\cos\alpha_1$			
8	$B . 10^4$ for ϕ_1			
9	$C . 10^8$ for ϕ_1			
10	$D . 10^8$ for ϕ_1			
11	$E . 10^{12}$ for ϕ_1			
12	5 . 6			
13		5 . 7 . 8	±	
14		$12^2 . 9$	±	
15		$12^2 . 11 . 13 . 10^{-4}$	±	
16		$13 + 14 + 15 = \Delta'\phi''$		
17		$(16 . 10^{-4})^2 . 10$	±	
18		$16 + 17 = \Delta\phi''$		
19		$\Delta\phi$	±	
20		$\Delta\phi / 2$		
21		ϕ_1	±	
22		ϕ_m		
23		$\Delta\phi / 2$		
24	LATITUDE	ϕ_2	±	
25	$A . 10^4$ for ϕ_2			
26	$\cos\phi_2$			
27		$25 . 12 / 26 = \Delta\lambda''$		
28		$\Delta\lambda$	±	
29		λ_1	±	
30	LONGITUDE	λ_2	±	
31	sin 22			
32	cos 20			
33		$27 . 31/32 = \Delta\alpha''$		
34		$\Delta\alpha$	±	
35		$\alpha_1 \pm 180°$		
36	AZIMUTH	α_2		

$$\Delta'\phi'' = B_1 . s . \cos\alpha_1 - C_1 . s^2 . \sin^2\alpha_1 - (B_1 . s . \cos\alpha_1)\, E_1 . s^2 . \sin^2\alpha_1$$

$$\Delta\phi'' = \Delta'\phi'' - D_1 . (\Delta'\phi'')^2$$

$$\Delta\lambda'' = A_2 . s . \sin\alpha_1 / \cos\phi_2$$

$$\Delta\alpha'' = \Delta\lambda'' . \sin\phi_m \;/ \cos(\Delta\phi/2)$$

SIGN CONVENTION

Latitude ϕ :	North	+
	South	−
Longitude λ :	East	+
	West	−

Determine the signs of each term from the diagrams below and form sums ignoring signs in the formulae above.

NORTHERN HEMISPHERE

14 & 17 are −

$\alpha_1 = 0°$

13	+	+
15	−	−
$\Delta\phi$	+	+
$\Delta\lambda$	−	+
$\Delta\alpha$	−	+

270° ——— 90°

13	−	−
15	+	+
$\Delta\phi$	−	−
$\Delta\lambda$	−	+
$\Delta\alpha$	−	+

$\alpha_1 = 180°$

SOUTHERN HEMISPHERE

14 & 17 are +

$\alpha_1 = 0°$

13	+	+
15	−	−
$\Delta\phi$	+	+
$\Delta\lambda$	−	+
$\Delta\alpha$	+	−

270° ——— 90°

13	−	−
15	+	+
$\Delta\phi$	−	−
$\Delta\lambda$	−	+
$\Delta\alpha$	+	−

$\alpha_1 = 180°$

Computed ____________________

Checked ____________________

Date ____________________

LATITUDE AND LONGITUDE FROM DISTANCE AND AZIMUTH

SIMPLIFIED PUISSANT'S FORMULAE

From STATION 1 BUNINYONG

To STATION 2 FLINDERS PEAK

$$\Delta'\phi'' = B_1 . s . \cos\alpha_1 - C_1 . s^2 . \sin^2\alpha_1 - (B_1 . s . \cos\alpha_1)\, E_1 . s^2 . \sin^2\alpha_1$$

$$\Delta\phi'' = \Delta'\phi'' - D_1 . (\Delta'\phi'')^2$$

$$\Delta\lambda'' = A_2 . s . \sin\alpha_1 / \cos\phi_2$$

$$\Delta\alpha'' = \Delta\lambda'' . \sin\phi_m \ / \cos(\Delta\phi/2)$$

1	ϕ_1	-37° 39 15" 56		
2	λ_1	+143 55 30 63		
3	α_1	127 10 27 1		
4	s (metres)	54972 2		

5	$s . 10^{-4}$	5 497 22		
6	$\sin\alpha_1$	0 796 802		
7	$\cos\alpha_1$	0 604 240		
8	$B . 10^4$ for ϕ_1	324 353		
9	$C . 10^8$ for ϕ_1	0 195 95		
10	$D . 10^8$ for ϕ_1	2 360 5		
11	$E . 10^{12}$ for ϕ_1	0 011 38		
12	5 . 6	4 380 196		
13		5 . 7 . 8	±	- 1077 38
14		$12^2 . 9$	±	+ 3 76
15		$12^2 . 11 . 13 . 10^{-4}$	±	+ 0 02
16		$13 + 14 + 15 = \Delta'\phi''$		- 1073 60
17		$(16 . 10^{-4})^2 . 10$	±	+ 0 03
18		$16 + 17 = \Delta\phi''$		- 1073 57
19		$\Delta\phi$	±	- 17 53 57
20		$\Delta\phi / 2$		- 08 56 78
21		ϕ_1	±	- 37 39 15 56
22		ϕ_m		- 37 48 12 34
23		$\Delta\phi / 2$		- 08 56 79
24	LATITUDE	ϕ_2	±	-37° 57' 09" 13

25	$A . 10^4$ for ϕ_2	322 983		
26	$\cos\phi_2$	0 788 521		
27		$25 . 12 / 26 = \Delta\lambda''$		1794 15
28		$\Delta\lambda$	±	+ 29 54 15
29		λ_1	±	+ 143 55 30 63
30	LONGITUDE	λ_2	±	+ 144° 25' 24" 78

31	sin 22	0 612 954		
32	cos 20	0 999 997		
33		$27 . 31 / 32 = \Delta\alpha''$		1099 7
34		$\Delta\alpha$	±	- 18 19 7
35		$\alpha_1 \pm 180°$		307 10 27 1
36	AZIMUTH	α_2		306° 52' 07" 4

Computed

Checked

Date Wed 30 Jul/69

SIGN CONVENTION

Latitude ϕ :	North	+
	South	−
Longitude λ :	East	+
	West	−

Determine the signs of each term from the diagrams below and form sums ignoring signs in the formulae above.

NORTHERN HEMISPHERE

14 & 17 are −

$\alpha_1 = 0°$

13	+	+
15	−	−
$\Delta\phi$	+	+
$\Delta\lambda$	−	+
$\Delta\alpha$	−	+

270° — 90°

13	−	−
15	+	+
$\Delta\phi$	−	−
$\Delta\lambda$	−	+
$\Delta\alpha$	−	+

$\alpha_1 = 180°$

SOUTHERN HEMISPHERE

14 & 17 are +

$\alpha_1 = 0°$

13	+	+
15	−	−
$\Delta\phi$	+	+
$\Delta\lambda$	−	+
$\Delta\alpha$	+	−

270° — 90°

13	−	−
15	+	+
$\Delta\phi$	−	−
$\Delta\lambda$	−	+
$\Delta\alpha$	+	−

$\alpha_1 = 180°$

6.4 DISTANCE AND AZIMUTH FROM LATITUDE AND LONGITUDE: SIMPLIFIED MID-LATITUDE FORMULAE

6.4.1 The following formulae are much simpler than those described in *5.6* and are accurate to about 0·3 metres at distances of 40 kilometres:

$$\begin{aligned}
\Delta a'' &= \Delta\lambda'' \sin \phi_m \\
\tan (a + \Delta a/2) &= B_m\, \Delta\lambda'' \cos \phi_m \,/\, (A_m\, \Delta\phi'') \\
\cot (a + \Delta a/2) &= A_m\, \Delta\phi'' \,/\, (B_m\, \Delta\lambda'' \cos \phi_m) \\
s &= \Delta\phi''/[B_m \cos (a + \Delta a/2)] \\
s &= \Delta\lambda'' \cos \phi_m \,/\, [A_m \sin (a + \Delta a/2)]
\end{aligned}$$

6.4.2 See the computation form and example at *6.5*. Enter the data on lines 1, 2, 8 and 9, determine 3 to 7, 10 and 11. Using 5 as argument, extract the values for 17 and 18 from TM 5-241-33. Enter 12 and 16 and calculate 13 to 15 and 19 to 21. Select the appropriate line 21 and calculate 22, at the same time entering the sine and cosine in the two lines numbered 28. The quadrant of 22 may be determined by carrying algebraic signs on 3 and 10 or may be derived by inspection of the coordinates of stations 1 and 2. Compute the azimuth, reverse azimuth and distance. If there is a difference in the distance, check first for gross error and then adopt the value derived from the larger of sin 22 and cos 22 in line 28.

6.5

DISTANCE AND AZIMUTH FROM LATITUDE AND LONGITUDE

SIMPLIFIED MID-LATITUDE FORMULAE

STATION 1 ______________________ STATION 2 ______________________

1	ϕ_1	
2	ϕ_2	
3	$\phi_2 - \phi_1 = \Delta\phi$	
4	$\Delta\phi/2$	
5	$\phi_1 + \Delta\phi/2 = \phi_m$	
6	$\phi_m + \Delta\phi/2 = \phi_2$ (Check)	
7	$\Delta\phi''$	
8	λ_1	
9	λ_2	
10	$\lambda_2 - \lambda_1 = \Delta\lambda$	
11	$\Delta\lambda''$	
12	sin 5	
13	$11 . 12 = \Delta\alpha''$	
14	$\Delta\alpha$ ±	
15	$\Delta\alpha/2$	

16	cos 5	
17	$A . 10^4$ for ϕ_m	
18	$B . 10^4$ for ϕ_m	
19	11.16.18	
20	7 . 17	
21	$19/20 = \tan(\alpha + \Delta\alpha/2)$	
	OR (select the smaller)	
21	$20/19 = \cot(\alpha + \Delta\alpha/2)$	
22	$\alpha + \Delta\alpha/2$	
23	−15 ±	
24	**Azimuth α_1**	
25	24 ±180°	
26	14 ±	
27	**Reverse Azimuth α_2**	
28	cos 22	
29	$[7/(18 . 28)] . 10^4 = s$	
	And	
28	sin 22	
29	$[(11.16)/(17.28)]\ 10^4 = s$	

Adopt 29 from larger 28

SIGN CONVENTION

Latitude ϕ	NORTH	+
	SOUTH	−
Longitude λ	EAST	+
	WEST	−

NORTHERN HEMISPHERE

When STATION 2 is EAST / WEST of STATION 1 $\Delta\alpha$ is + / −

SOUTHERN HEMISPHERE

When STATION 2 is EAST / WEST of STATION 1 $\Delta\alpha$ is − / +

$$\Delta\alpha'' = \Delta\lambda'' . \sin\phi_m$$

$$\tan(\alpha + \Delta\alpha/2) = B_m . \Delta\lambda'' . \cos\phi_m \,/\, (A_m . \Delta\phi'')$$

$$\cot(\alpha + \Delta\alpha/2) = A_m . \Delta\phi'' \,/ (B_m . \Delta\lambda'' . \cos\phi_m)$$

$$s = \Delta\phi''/[B_m . \cos(\alpha + \Delta\alpha/2)]$$

$$s = \Delta\lambda'' . \cos\phi_m \,/\, [A_m . \sin(\alpha + \Delta\alpha/2)]$$

Computed ______________________

Checked ______________________

Date ______________________

DISTANCE AND AZIMUTH FROM LATITUDE AND LONGITUDE

SIMPLIFIED MID-LATITUDE FORMULAE

STATION 1 BUNINYONG

STATION 2 FLINDERS PEAK

1	ϕ_1	– 37 39 15·56
2	ϕ_2	– 37 57 09·13
3	$\phi_2 - \phi_1 = \Delta\phi$	– 17 53 57
4	$\Delta\phi/2$	08 56·78
5	$\phi_1 + \Delta\phi/2 = \phi_m$	37 48 12·34
6	$\phi_m + \Delta\phi/2 = \phi_2$ (Check)	37 57 09·12
7	$\Delta\phi''$	1073·57
8	λ_1	+143 55 30·63
9	λ_2	+144 25 24·79
10	$\lambda_2 - \lambda_1 = \Delta\lambda$	+ 29 54·16
11	$\Delta\lambda''$	1794·16
12	sin 5	0·612 954
13	11 . 12 = $\Delta\alpha''$	1099·7
14	$\Delta\alpha$ ±	–00 18 19·7
15	$\Delta\alpha/2$	–00 09 09·8

16	cos 5	0·790 118
17	A . 10^4 for ϕ_m	322·9854
18	B . 10^4 for ϕ_m	324·3443
19	11 . 16 . 18	459 789·4
20	7 . 17	346 747·4
21	19/20 = $\tan(\alpha+\Delta\alpha/2)$	
	OR (select the smaller)	
21	20/19 = $\cot(\alpha+\Delta\alpha/2)$	0·754 143
22	$\alpha + \Delta\alpha/2$	127 01 17·5
23	–15 ±	+ 09 09·8
24	**Azimuth** α_1	127° 10′ 27″·3
25	24 ± 180°	307 10 27·3
26	14 ±	– 18 19·7
27	**Reverse Azimuth** α_2	306° 52′ 07″·6
28	cos 22	0·602 115
29	[7/(18 . 28)] . 10^4 = s	54 972·4
	And	
28	sin 22	0·798 409
29	[(11 . 16)/(17 . 28)] 10^4 = s	54 972·4

Adopt 29 from larger 28

SIGN CONVENTION

Latitude ϕ	NORTH	+
	SOUTH	–
Longitude λ	EAST	+
	WEST	–

NORTHERN HEMISPHERE

When STATION 2 is EAST / WEST of STATION 1 $\Delta\alpha$ is + / –

SOUTHERN HEMISPHERE

When STATION 2 is EAST / WEST of STATION 1 $\Delta\alpha$ is – / +

$$\Delta\alpha'' = \Delta\lambda'' . \sin\phi_m$$

$$\tan(\alpha+\Delta\alpha/2) = B_m . \Delta\lambda'' . \cos\phi_m \,/\, (A_m . \Delta\phi'')$$

$$\cot(\alpha+\Delta\alpha/2) = A_m . \Delta\phi'' \,/(B_m . \Delta\lambda'' . \cos\phi_m)$$

$$s = \Delta\phi''/[B_m . \cos(\alpha+\Delta\alpha/2)]$$

$$s = \Delta\lambda'' . \cos\phi_m \,/\, [A_m . \sin(\alpha+\Delta\alpha/2)]$$

Computed

Checked

Date Wed 30 Jul/69

6.6 SIMPLIFIED ZONE TO ZONE TRANSFORMATION

6.6.1 The method used here is similar to that described in *5.12* except that the terms involving H and J have been omitted. The symbols and tabulated functions are defined in *5.12.4*.

6.6.2 The following formulae provide coordinates accurate to 1 metre where the point is within approximately 10 kilometres of a zone boundary. Beyond this the accuracy falls off very rapidly but yields values within 10 metres at approximately 40 kilometres from a zone boundary:

$$E_2 = 500\,000 - E'_z + (E'_1 - E'_z) \cos 2\gamma_z - (N_1 - N_z) \sin 2\gamma_z$$
$$N_2 = N_z + (N_1 - N_z) \cos 2\gamma_z + (E'_1 - E'_z) \sin 2\gamma_z$$

6.6.3 Values of E'_z, $\sin 2\gamma_z$ and $(1-\cos 2\gamma_z)\ 10^4$ are tabulated in Annex E. Enter the N_z column with N_1 and extract the closest listed value of N_z and the required functions without interpolation.

6.6.4 See the computation form and example at *6.7*. Enter the data on lines 5 and 15. Extract the tabulated values from Annex E for lines 1, 2, 8 and 16. Always enter 8 as positive. Calculate 7, 9 and 17. Note that the differences are arithmetic the signs being determined from the sign convention. Determine the quadrant of θ_z and enter 3 and 4. Compute 10 to 14 and 18 to 20. The example illustrates the maximum number of decimal places required in all cases.

6.7

SIMPLIFIED ZONE TO ZONE TRANSFORMATION

AUSTRALIAN MAP GRID

ZONE__________(1) **To ZONE**__________(2) **E to W**
W to E

STATION ____________________

$$E_2 = 500000 - E'_z + (E'_1 - E'_z)\cos 2\gamma_z - (N_1 - N_z)\sin 2\gamma_z$$

$$N_2 = N_z + (N_1 - N_z)\cos 2\gamma_z + (E'_1 - E'_z)\sin 2\gamma_z$$

1	$\sin 2\gamma_z$	
2	$(1-\cos 2\gamma_z).10^4$	
3	9.10^{-4}	
4	17.10^{-4}	

5	E_1		
6	False Origin		500 000.000
7	Difference 5 and 6 = E'_1		
8	E'_z		+
9	Difference 7 and 8	±	
10	1.17	±	
11	2.3	±	
12	Sum 8 to 11 = E'_2	±	
13	False Origin		500 000.000
14	12+13= **EASTING** E_2		
15	N_1		
16	N_z		
17	Difference 15 and 16	±	
18	1.9	±	
19	2.4	±	
20	Sum 16 to 19 = **NORTHING** N_2		

SIGN CONVENTION

SOUTHERN HEMISPHERE

W Zone to E Zone			**E Zone to W Zone**	
12 is −		12	12 is +	
$\theta_z = 0°$			$\theta_z = 0°$	
+	−	9	−	+
+	+	10	+	+
−	+	11	+	−
+	+	17	+	+
−	+	18	+	−
−	−	19	−	−
270°	Z 90°		270°	Z 90°
+	−	9	−	+
−	−	10	−	−
−	+	11	+	−
−	−	17	−	−
−	+	18	+	−
+	+	19	+	+
$\theta_z = 180°$			$\theta_z = 180°$	

NOTE: Determine the quadrant of θ_z by inspection

Computed ____________________

Checked ____________________

Date ____________________

SIMPLIFIED ZONE TO ZONE TRANSFORMATION

AUSTRALIAN MAP GRID

ZONE 55 (1) To ZONE 54 (2) E to W / W to E STATION BUNINYONG

$$E_2 = 500000 - E'_z + (E'_1 - E'_z)\cos 2\gamma_z - (N_1 - N_z)\sin 2\gamma_z$$

$$N_2 = N_z + (N_1 - N_z)\cos 2\gamma_z + (E'_1 - E'_z)\sin 2\gamma_z$$

1	$\sin 2\gamma_z$	0·064 10
2	$(1-\cos 2\gamma_z).10^4$	20·568
3	9.10^{-4}	0·694
4	17.10^{-4}	1·040

5	E_1		228 742·1
6	False Origin		500 000.000
7	Difference 5 and 6 = E'_1		271 257·9
8	E'_z		+264 315·1
9	Difference 7 and 8	±	− 6 942·8
10	1.17	±	+ 667·0
11	2.3	±	+ 14·3
12	Sum 8 to 11 = E'_2	±	+258 053·6
13	False Origin		500 000.000
14	12+13 = **EASTING** E_2		758 053·6
15	N_1		5 828 074·2
16	N_z		5 817 669·3
17	Difference 15 and 16	±	+ 10 404·9
18	1.9	±	+ 445·0
19	2.4	±	− 21·4
20	Sum 16 to 19 = **NORTHING** N_2		5 828 497·8

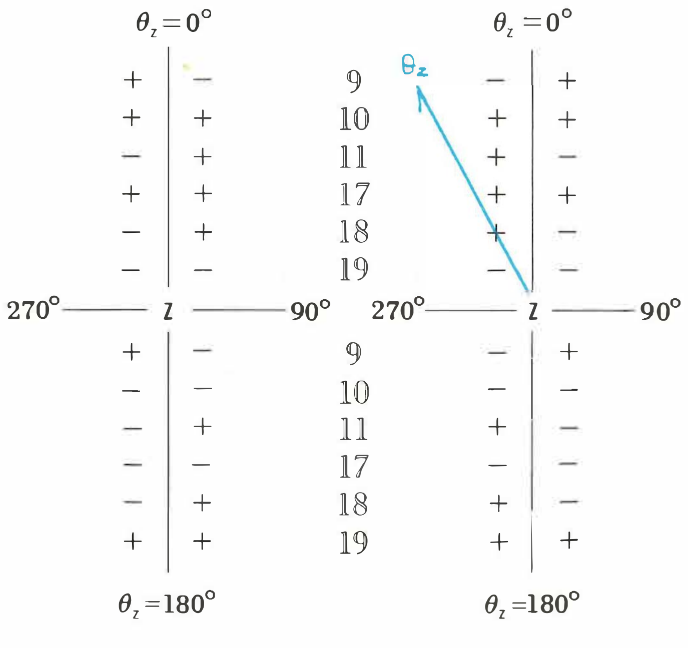
SIGN CONVENTION

SOUTHERN HEMISPHERE

W Zone to E Zone 12 is −

12

E Zone to W Zone 12 is +



	W Zone to E Zone, 270° side	W Zone to E Zone, 90° side		E Zone to W Zone, 270° side	E Zone to W Zone, 90° side
$\theta_z = 0°$					
	+	−	9	−	+
	+	+	10	+	+
	−	+	11	+	−
	+	+	17	+	+
	−	+	18	+	−
	−	−	19	−	−
$\theta_z = 180°$					
	+	−	9	−	+
	−	−	10	−	−
	−	+	11	+	−
	−	−	17	−	−
	−	+	18	+	−
	+	+	19	+	+

NOTE: Determine the quadrant of θ_z by inspection

Computed

Checked

Date 14 Sep 69

6.8 SPHEROID TO GRID – SIMPLIFIED TRANSFORMATION AND GRID CONVERGENCE

6.8.1 This computation uses Volume 1 of the US Army Tables TM 5-241-32/1. Given the latitude and longitude of a point, then its zone number, easting, northing and grid convergence can be determined. The formulae used here are the same as given in paragraphs *5.8.3, 5.8.4* and *5.8.5* but exclude the graphed terms and the small correction Δ^2 (IV).

6.8.2 The following formulae provide grid coordinates accurate to 0·3 metre and grid convergence to 1″, which is adequate for many purposes:

$E' = (IV)\ p + (V)\ p^3$

$E = E' + 500\,000$ metres, where E′ is taken negative west of a central meridian.

$N' = (I) + (II)\ p^2 + (III)\ p^4$

$N = N'$, north of the equator.

$N = N' + 10\,000\,000$ metres south of the equator, where N′ is taken negative.

$\gamma'' = (XII)\ p + (XIII)\ p^3$

6.8.3 See the example on page 59 using the form *5.9.* Compute $\omega = \lambda - \lambda_0$ and $p = \omega''.\ 10^{-4}$. Then ω and p are always taken positive, so that the sign of each term is the same as the sign of the tabulated function.

6.8.4 Tabular values for (I), (II), (IV), (XII) and (XIII) are always positive, (III) is negative beyond latitude 65° 56′ and (V) is negative between latitudes 45° 03′ and 80°. Interpolate in the tables in a self-checking manner as described in paragraph *5.3.1.* The tabular values and increments need not then be recorded. The last two figures of tabular values can be dropped.

6.8.5 The sign of γ is taken from the diagram on the form. Note that the sign of γ used for the AMG is the opposite of C used in the US Army Tables.

6.8.6 For lower order work:

$\gamma'' = (XII)\ p$

$= -(\lambda - \lambda_0) \sin \phi$

On the Zone boundary the error in this simple formula is less than 4″.

TRANSFORMATION OF COORDINATES FROM GEOGRAPHIC TO GRID

AUSTRALIAN MAP GRID

ZONE 55 STATION BUNINYONG

Zone	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Central Meridian	87°	93°	99°	105°	111°	117°	123°	129°	135°	141°	147°	153°	159°	165°	171°	177°E

1	Latitude ϕ		37 39 15·557
2	Longitude λ		143 55 30·633
3	Central Meridian λ_0		147
4	$\omega = \lambda - \lambda_0$		− 03 04 29·367
5	ω''		
6	(I) Tabular value		
7	Increment		
8	(I) for ϕ		4 167 476·9
9	(II) Tabular value		
10	Increment		
11	(II) for ϕ		3 628·4
12	(III) for ϕ	±	+ 2·0
13	(IV) Tabular value		
14	Increment		
15	Δ^2 (IV)		
16	(IV) for ϕ		245 022·7
17	(V) Tabular value		
18	Increment		
19	(V) for ϕ	±	+ 25·6
20	(XII) Tabular value		
21	Increment		6109·0
22	(XII) for ϕ		6109·0
23	(XIII) for ϕ		3·0

24	$p = \cdot 0001.\omega''$	+	1·106 9367
25	p^2	+	1·225 31
26	p^3	+	1·356
27	p^4	+	1·50
28	16.24	+	271 224·6
29	19.26	±	+ 33·4
30	B_5 for ϕ, ω	±	
31	$28 + 29 + 30 = E'$	±	− 271 258·0
32	False Origin	+	500 000 ·000
33	$31 + 32 =$ **Easting** E		228 742·0
34	8	+	4 167 476·9
35	11.25	+	4 445·9
36	12.27	±	+ 3·0
37	A_6 for ω	±	
38	$34+35+36+37 = N'$	±	− 4 171 925·8
39	Southern Hemisphere		+ 10 000 000 ·000
40	$38+39 =$ **Northing** N		5 828 074·2
41	22.24	+	6762
42	23.26	+	4
43	C_5 for ω	+	
44	$41 + 42 + 43 = \gamma''$		− 6766
45	**Grid Convergence** γ	±	− 01° 52′ 46″

$E' = IV.p + V.p^3 + B_5$

$E = 500{,}000 + E'$

$N' = I + II.p^2 + III.p^4 + A_6$

$N = N'$ north of equator

$N = 10{,}000{,}000 + N'$ south of equator

$\gamma = XII.p + XIII.p^3 + C_5$

SIGN CONVENTION

NOTE: 1. p & ω are always taken positive

2. (III) is negative beyond latitude 65°56′ N & S

(V) is negative between latitudes 45°03′ & 80° N & S

B_5 is negative between latitudes 27°57′ & 76°40′ N & S

A_6 is negative between latitudes 50° & 68° N & S

3. $\gamma = -C$ used in US Army tables

Grid bearing = Azimuth + γ = Azimuth − C

NORTHERN HEMISPHERE				SOUTHERN HEMISPHERE	
λ_0				λ_0	
−	+	E′		−	+
+	+	N′		−	−
+	−	γ		−	+

Computed R. N'dell

Checked

Date 17 Dec 69

6.9 GRID TO SPHEROID: SIMPLIFIED TRANSFORMATION AND GRID CONVERGENCE

6.9.1 This computation uses Volume 2 of the US Army Tables, TM 5-241-32/2. Given the zone number, easting and northing of a point, then its latitude, longitude and grid convergence can be determined. The formulae used here are the same as given in paragraphs *5.10.2* to *5.10.6* but exclude the graphed terms and the small correction Δ^2 (IX).

6.9.2 The following formulae provide geographic coordinates accurate to $0''.01$ in latitude and longitude and grid convergence to $1''$, which is adequate for many purposes.

E' = E – 500 000 metres.
N' = N – 10 000 000 metres, south of the equator.
N' = N, north of the equator.
$\phi = \phi' - (\text{VII})\, q^2 + (\text{VIII})\, q^4$
$\omega'' = (\text{IX})\, q - (\text{X})\, q^3$
$\lambda = \lambda_0 + \omega$
$\gamma'' = (\text{XV})\, q - (\text{XVI})\, q^3$

6.9.3 See the example on page 61 using the form *5.11.* Compute $q = E'\ 10^{-6}$ and ϕ' by reverse interpolation in table (I) by the method explained in paragraph *5.3.2.* Then q and ϕ' are always taken positive.

6.9.4 Interpolate in the tables for (VII), (VIII), (IX), (X), (XV) and (XVI) in a self-checking manner as described in paragraph *5.3.1.* The tabular values and increments need not then be recorded. The last two figures of tabular values and of all other calculations may be dropped.

6.9.5 In the southern hemisphere the computed latitude is taken as negative. ω is also taken as negative when west of a central meridian. The sign of γ is taken from the diagram on the form. Note that the sign of γ used for the AMG is the opposite of C used in the US Army Tables.

TRANSFORMATION OF COORDINATES FROM GRID TO GEOGRAPHIC

AUSTRALIAN MAP GRID

ZONE 55

STATION BUNINYONG

Zone	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Central Meridian	87°	93°	99°	105°	111°	117°	123°	129°	135°	141°	147°	153°	159°	165°	171°	177°E

1	**Easting** E		228 742·1
2	**False Origin**	−	500 000 ·000
3	E′	±	- 271 257·9
4	**Northing** N		5 828 074·2
5	**Southern Hemisphere**	−	10 000 000 ·000
6	N′	±	- 4 171 925·8
7	(I) Tabular value		
8	6 − 7		
9	ϕ' for N′		37° 41′ 39″92
10	(VII) Tabular value		
11	Increment		
12	(VII) for ϕ'		1963·9
13	(VIII) for ϕ'		27·2
14	(IX) Tabular value		
15	Increment		
16	Δ^2 (IX)	−	
17	(IX) for ϕ'		40 834·52
18	(X) Tabular value		
19	Increment		
20	(X) for ϕ'		367·2
21	(XV) Tabular value		
22	Increment		
23	(XV) for ϕ'		24 968
24	(XVI) for ϕ		325

25	$q = E'.10^{-6}$	+	0·271 2579
26	q^2	+	0·073 581
27	q^3	+	0·019 96
28	q^4	+	0·005 4
29	9	+	37 41 39·92
30	12 . 26	−	02 24·50
31	13 . 28	+	0·15
32	D_6 for q	−	
33	29 + 30 + 31 + 32 = ϕ **Latitude**	±	-37° 39′ 15·57
34	17 . 25	+	11076·69
35	20 . 27	−	7·33
36	E_5 for q	+	
37	34 + 35 + 36 = ω''		11069·36
38	ω	±	- 03 04 29·36
39	Central Meridian λ_o		147
40	38 + 39 = λ **Longitude**		+143° 55′ 30″64
41	23 . 25	+	6773
42	24 . 27	−	6
43	F_5 for q	+	
44	41 + 42 + 43 = γ''		6767
45	**Grid Convergence** γ	±	- 01° 52′ 47″

$q = E'.10^{-6}$

$\phi = \phi' - VII.q^2 + VIII.q^4 - D_6$

$\omega = IX.q - X.q^3 + E_5$

$\lambda = \lambda_o + \omega$

$\gamma = XV.q - XVI.q^3 + F_5$

SIGN CONVENTION

NOTE:
1. q & ϕ' are always taken positive
2. Δ^2(IX) is always negative
3. $\gamma = -C$ used in US Army tables

Grid Bearing = Azimuth + γ = Azimuth − C

NORTHERN HEMISPHERE λ_o			**SOUTHERN HEMISPHERE** λ_o	
−	+	E′	−	+
+	+	N′	−	−
−	+	ω	−	+
+	−	γ	−	+

Computed J Hill

Checked C Ford

Date 18 Dec 69

6.10 SCALE FACTORS

6.10.1 In order to convert a spheroidal distance s to a plane distance L, or to obtain the spheroidal distance from the plane or grid distance, it is necessary to calculate the line scale factor. Since a Transverse Mercator projection is conformal, the scale at any point is the same in all directions; but varies with the distance from the central meridian.

6.10.2 To minimise the scale factor over the whole of the zone width, a central scale factor $k_0 = 0{\cdot}999\,6$ is superimposed over all projected distances. After the application of this central scale factor, the point scale factor will vary from 0·999 6 on a central meridian to almost 1·001 0 on a zone boundary at the equator and to 1·000 0 on a zone boundary at latitude 56°.

6.10.3 **Point Scale Factor**

The rigorous equation for point scale factor given in paragraph *4.5.4* can be reduced to

$$k = k_0\ [1 + (E'^2/2r^2) + (E'^4/24r^4)]$$

This has been used in the US Army Tables in the form:

$$k = k_0\ [1 + (\mathrm{XVIII})\, q^2 + (\mathrm{XIX})\, q^4]$$

where $q = E'\ 10^{-6}$ and is always taken positive.

This equation is accurate to 1 part in 10 million. Omission of the last term results in an accuracy of 2 parts in 10 million.

6.10.4 The equation can be simplified further using the following approximation:

$$k = 0{\cdot}999\,6 + 0{\cdot}012\,3\ E'^2\ 10^{-12}$$

which is easy to remember. The formula is correct at latitude 34° and is accurate to 8 ppm at the equator.

Example:

Buninyong, Zone 55

$E = -\,271\,300$

$E'^2 = 0{\cdot}073\,60.10^{12}$

$k = 0{\cdot}999\,6 + (0{\cdot}012\,3.0{\cdot}073\,6)$

$= 1{\cdot}000\,505$

which is accurate to 2 ppm.

6.10.5 Point scale factors may be taken from either Annex G or the graph on the second page of Volumes 1 or 2 of the US Army Tables TM 5-241-32.

6.10.6 **Line Scale Factor**

The scale factor will in general vary from one end of a line to the other, and the method used to determine a scale factor for the whole line will depend upon the accuracy of the survey and the length of the line.

6.10.7 The equation for line scale factor given in paragraph *5.16.2* can be reduced to

$$K = K_0\ [1 + (E_1'^2 + E_1'\ E_2' + E_2'^2)/6r_m^2]$$

6.10.8 For single lines or traverses, line scale factors can be obtained anywhere on the AMG by using either:

(1) The point scale factor for the mean easting of the line or traverse. This procedure is accurate to 1 ppm in any line or traverse extending 33 kilometres in easting. OR

(2) The mean of the point scale factors at the extremities of the line or traverse. This procedure is accurate to 1 ppm in any line or traverse extending 16 kilometres in easting.

The accuracies stated above are independent of the location in the zone.

6.10.9 The technique explained in *5.20* can be used with these simplified formulae.

6.11 ARC-TO-CHORD CORRECTIONS

6.11.1 On the Zone boundary the arc-to-chord correction for a north-south line is about 0″85 per kilometre.

6.11.2 The formula for the arc-to-chord correction given in paragraph *5.16.2* can be written:

$$\delta_1'' = -\,(N_2 - N_1)\,(2E_1 + E_2 - 1{\cdot}5.10^6)/6r_m^2\ \sin 1''$$

This yields results correct to 0″01 for a line 30 kilometres long.

6.11.3 If the numerical difference between the arc-to-chord corrections at each end of the line can be neglected, the formula can be simplified to:

$$\delta_1'' = -\delta_2'' = -(N_2 - N_1)(E_2 + E_1 - 10^6)/4r_m^2 \sin 1''$$
$$= \Delta\beta/2$$

6.11.4 The function $1/4r_m^2 \sin 1''$ has a value of $0{\cdot}127.10^{-8}$ anywhere on the Australian Map Grid. This formula reduces to:

$$\delta_1'' = -\delta_2'' = -(N_2 - N_1)(E_2 + E_1 - 10^6)(0{\cdot}127.10^{-8})$$

The error in this simplified formula is 0"·6 on the test line.

6.11.5 The technique explained in *5.20* can be used with these simplified formulae.

7 Grid References

7.1 The Universal Transverse Mercator Grid, of which the Australian Map Grid is part, covers the whole world except the polar regions beyond latitude 84°N and 80°S. A grid reference uniquely defines any point on the grid.

7.2 Maps published on the Australian Map Grid at scales of 1:250 000 and larger show:

7.2.1 A series of straight grid lines covering the map in squares – see Annex H. Maps at scales of 1:100 000 and larger have grid lines at intervals of 1 kilometre. At 1:250 000 the grid lines are at intervals of 10 kilometres.

7.2.2 An explanation in the margin of how to give a grid reference – see Annex I.

7.2.3 A box in the margin showing the 1:100 000 map sheet numbers.

7.3 SIX-FIGURE REFERENCES

7.3.1 Six-figure references may be used on any map where the grid lines are at intervals of 1 kilometre.

7.3.2 Instructions for giving a six-figure reference are shown in notes 2 to 5 in the top box of Annex I. The six-figure reference of 'The Lion' is 973523. Other examples can be found on maps published on the Australian Map Grid or the Universal Transverse Mercator Grid where the grid interval is 1 kilometre.

7.3.3 A six-figure reference defines a point to the nearest 100 metres. Points with an identical reference recur every 100 kilometres. For a unique reference, additional letters and figures are required – see *7.5* and *7.6*.

7.4 FOUR-FIGURE REFERENCE

7.4.1 On maps where the grid lines are at intervals of 10 kilometres, only four-figure references may be used.

7.4.2 Instructions for giving a four-figure reference are shown in notes 2 to 5 of the lower box in Annex I. The four-figure reference for 'The Lion' read from a map at a scale of 1:250 000 would be 9752.

7.4.3 A four-figure reference defines a point to the nearest kilometre. Four-figure references can also be used with maps at larger scales when giving a reference to a larger object, such as a lake or a city, for which a six-figure reference would be inappropriate. Four-figure references, like six-figure references, recur identically every 100 kilometres.

7.5 GRID REFERENCES – THE INTERNATIONAL SYSTEM

7.5.1 As six-figure and four-figure references recur at 100 kilometre intervals, it is necessary, for a unique definition, to precede them with further letters and figures.

7.5.2 Every 100 kilometre square is given a pair of letters. These letters are given in the grid reference box in the margin of the map. Thus 'The Lion', in Annex H, is shown in Annex I to be in the 100 kilometre square which has the letters MU. A more specific reference for 'The Lion' is MU973523. A single map sheet may contain parts of two or even four different 100 kilometre squares. The letters for each part are clearly indicated in the grid reference box, and are also indicated on the face of the map near the boundaries of 100 kilometre squares.

7.5.3 In military practice, six-figure references are always preceded by these two letters defining the 100 kilometre square, and the letters and figures are written consecutively without a gap.

7.5.4 The letters for the 100 kilometre squares in the area covered by the Australian Map Grid are shown in Annex J.

7.5.5 A grid reference of this type is still inadequate for a unique definition over the whole world. Similar references can recur at intervals of 1 000 kilometres. At the top of the left half of the grid references boxes in Annex I will be found a group of two figures and a letter called the *Grid Zone Designation.* For 'The Lion', the Grid Zone Designation is 56J. The complete grid reference for 'The Lion' is thus 56JMU973523, and no other point on Earth has this same reference. However, the Grid Zone Designation is seldom needed and is usually omitted.

7.5.6 The Grid Zone Designation for the Australian Map Grid area can be obtained from Annex A or from Annex J. The zone numbers always precede the letter defining the belt of latitude.

7.6 GRID REFERENCE – ALTERNATIVE SYSTEM

7.6.1 An alternative system which is preferred by some civilian map users is to identify the map and to quote a six-figure or four-figure reference. For example, 'The Lion' which lies on 1:100 000 map sheet 9441 could be identified by:

9441 – 973523

7.6.2 When using maps at larger scales, it is not necessary to quote the sub-division of the 1:100 000 sheet.

7.7 SPECIAL MAP SHEETS – RISK OF DUPLICATE SIX-FIGURE REFERENCES

If a map is published which straddles a grid zone boundary, identical six-figure references can occur on the one map sheet. Standard map sheets in the Australian Map Grid area do not cross zone boundaries, but should a special map be issued which does cross a boundary, this possibility should be borne in mind. If there is a risk of duplication, six-figure references must be preceded by the 100 kilometre square identification, and by the number of the standard 1:100 000 map sheet area in which the point lies.

7.8 GRID COORDINATES

In lists of coordinates for survey stations, surveyors usually quote Australian Map Grid coordinates by giving first the grid zone, for which zone numbers are listed along the bottom of Annex J, followed by the eastings of the point and then the northings, both if necessary given in millimetres. Thus for 'The Lion' we might find listed:

Station	Zone	Eastings	Northings
LION	56	497 346·612	6 852 369·405

Grid coordinates are essential when the highest precision is required, but the equivalent six-figure reference 973523 is usually sufficient.

RIGOROUS NUMERICAL VALUES FOR THE TEST LINE COMPUTED FROM REDFEARN'S AND ROBBINS'S FORMULAE

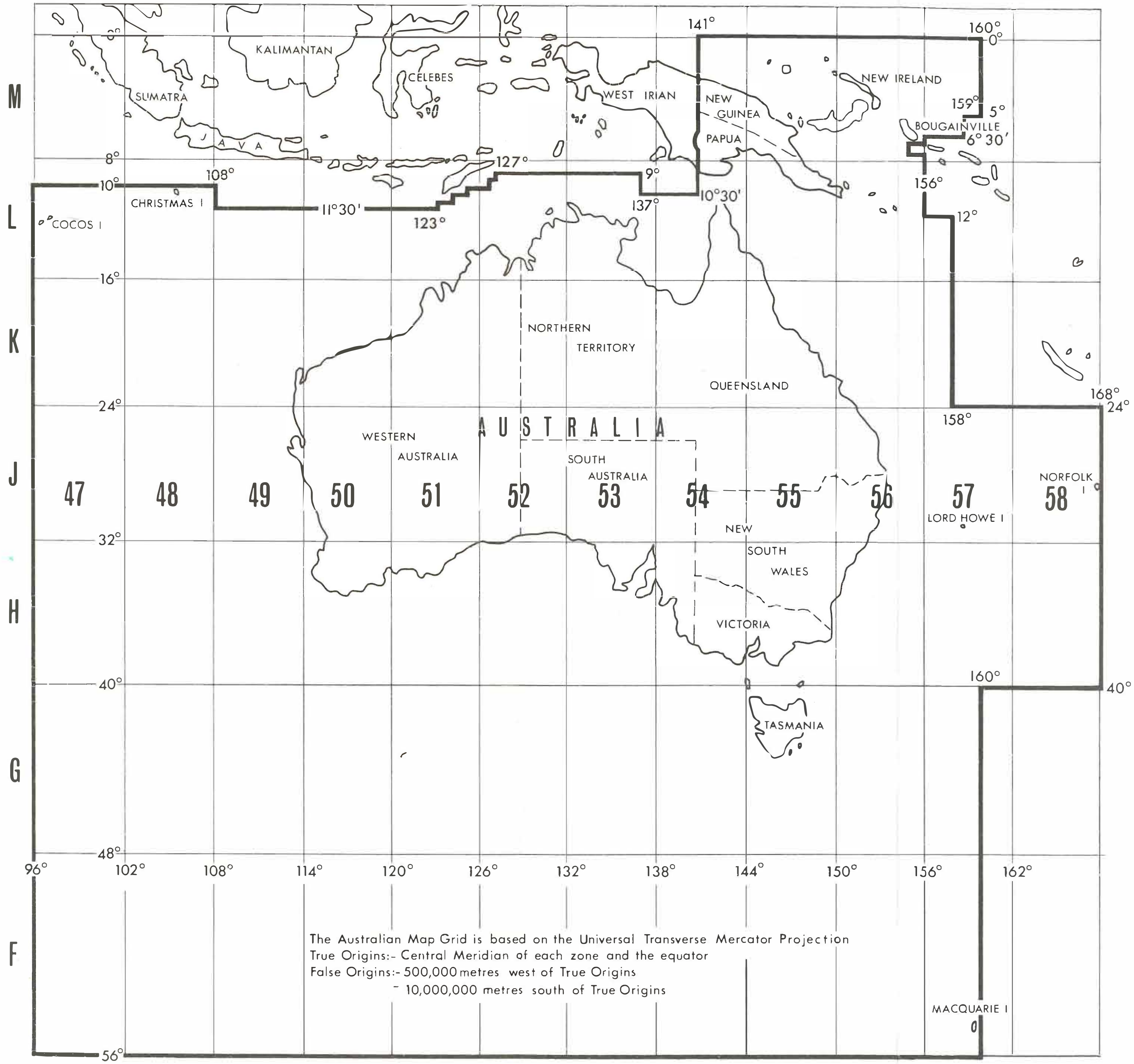
	Zone 54		*Zone 55*	
Station:	*Buninyong*	*Flinders Peak*	*Buninyong*	*Flinders Peak*
Latitude	– 37° 39′ 15″.557 1	– 37° 57′ 09″.128 8		
Longitude	+ 143 55 30·633 0	+ 144 25 24·786 6		
Easting	758 053·090	800 817·407	228 742·077	273 629·436
Northing	5 828 496·974	5 793 905·650	5 828 074·208	5 796 305·236
Azimuth	127° 10′ 27″.08	306° 52′ 07″.34	127° 10′ 27″.08	306° 52′ 07″.34
Grid Convergence	+ 1 47 16·67	+ 2 06 25·53	– 1 52 46·36	– 1 35 06·76
Grid Bearing	**128 57 43·75**	**308 58 32·87**	**125 17 40·72**	**305 17 00·58**
Arc-to-Chord	+ 23·94	– 25·18	– 20·67	+ 19·47
Plane Bearing	**128 58 07·69**	**308 58 07·69**	**125 17 20·05**	**305 17 20·05**
Point Scale Factor	1·000 420 30	1·000 714 68	1·000 506 41	1·000 231 18
Meridian Distance	– 4 169 144·533	– 4 202 244·285		
Rho, ρ	6 359 277·924	6 359 600·684		
Nu, ν	6 386 142·439	6 386 250·478		
Spheroidal Distance	54 972·161		54 972·161	
Line Scale Factor	1·000 563 72		1·000 364 62	
Grid Distance and Plane Distance	**55 003·150**		**54 992·205**	
Meridian Convergence	18′ 19″.74		18′ 19″.74	
Line Curvature	49″.12		40″.14	

All computations have been done with terms rounded as follows:

Latitude and Longitude	0″.000 1
Angles, Azimuths and Bearings	0″.01
Distances	0·001 metres

ANNEX A

LIMITS OF THE AUSTRALIAN MAP GRID



The Australian Map Grid is based on the Universal Transverse Mercator Projection
True Origins:- Central Meridian of each zone and the equator
False Origins:- 500,000 metres west of True Origins
- 10,000,000 metres south of True Origins

Formulae for terms used in transformation between geographic and grid coordinates, tabulated and graphed in TM5-241-32/1 and 32/2 of the U S Army Tables.

Tabular terms.

$$(I) = m\ k_0$$

$$(II) = \nu\ k_0\ \sin\phi\ \cos\phi\ \sin^2 1''\ 10^8/2$$

$$(III) = \nu\ k_0\ \sin\phi\ \cos^3\phi\ \sin^4 1''\ 10^{16}(5 - t^2 + 9e'^2\cos^2\phi + 4e'^4\cos^4\phi)/24$$

$$(IV) = \nu\ k_0\ \cos\phi\ \sin 1''\ 10^4$$

$$(V) = \nu\ k_0\ \cos^3\phi\ \sin^3 1''\ 10^{12}(1 - t^2 + e'^2\cos^2\phi)/6$$

$$(VII) = (1 + e'^2\cos^2\phi)\ t\ 10^{12}/(2\nu^2 k_0^2 \sin 1'') = t\ 10^{12}/2r^2\sin 1''$$

$$(VIII) = (5 + 3t^2 + 6e'^2\cos^2\phi - 6e'^2\sin^2\phi - 3e'^4\cos^4\phi - 9e'^4\cos^2\phi\ \sin^2\phi)\,t\ 10^{24}/(24\,\nu^4 k_0^4 \sin 1'')$$

$$(IX) = 10^6/(\nu\ k_0\ \cos\phi\ \sin 1'')$$

$$(X) = (1 + 2t^2 + e'^2\cos^2\phi)10^{18}/(6\nu^3 k_0^3 \cos\phi\ \sin 1'')$$

$$(XII) = \sin\phi\ 10^4$$

$$(XIII) = \sin\phi\ \cos^2\phi\ \sin^2 1''(1 + 3e'^2\cos^2\phi + 2e'^4\cos^4\phi)10^{12}/3$$

$$(XV) = t\ 10^6/(\nu\ k_0\ \sin 1'')$$

$$(XVI) = (1 + t^2 - e'^2\cos^2\phi - 2e'^4\cos^4\phi)\ t\ 10^{18}/(3\nu^3 k_0^3\ \sin 1'')$$

$$(XVIII) = (1 + e'^2\cos^2\phi)10^{12}/(2\nu^2 k_0^2) = 10^{12}/2r^2$$

Terms shown as graphs.

$$A_6 = p^6 \nu\ k_0 \sin^6 1'' \sin\phi\ \cos^5\phi(61 - 58t^2 + t^4 + 270e'^2\cos^2\phi - 330e'^2\sin^2\phi)10^{24}/720$$

$$B_5 = p^5\ \nu\ k_0 \cos^5\phi\ \sin^5 1''(5 - 18t^2 + t^4 + 14e'^2\cos^2\phi - 58e'^2\sin^2\phi)10^{20}/120$$

$$C_5 = p^5 \sin\phi\ \cos^4\phi\ \sin^4 1''(2 - t^2)10^{20}/15$$

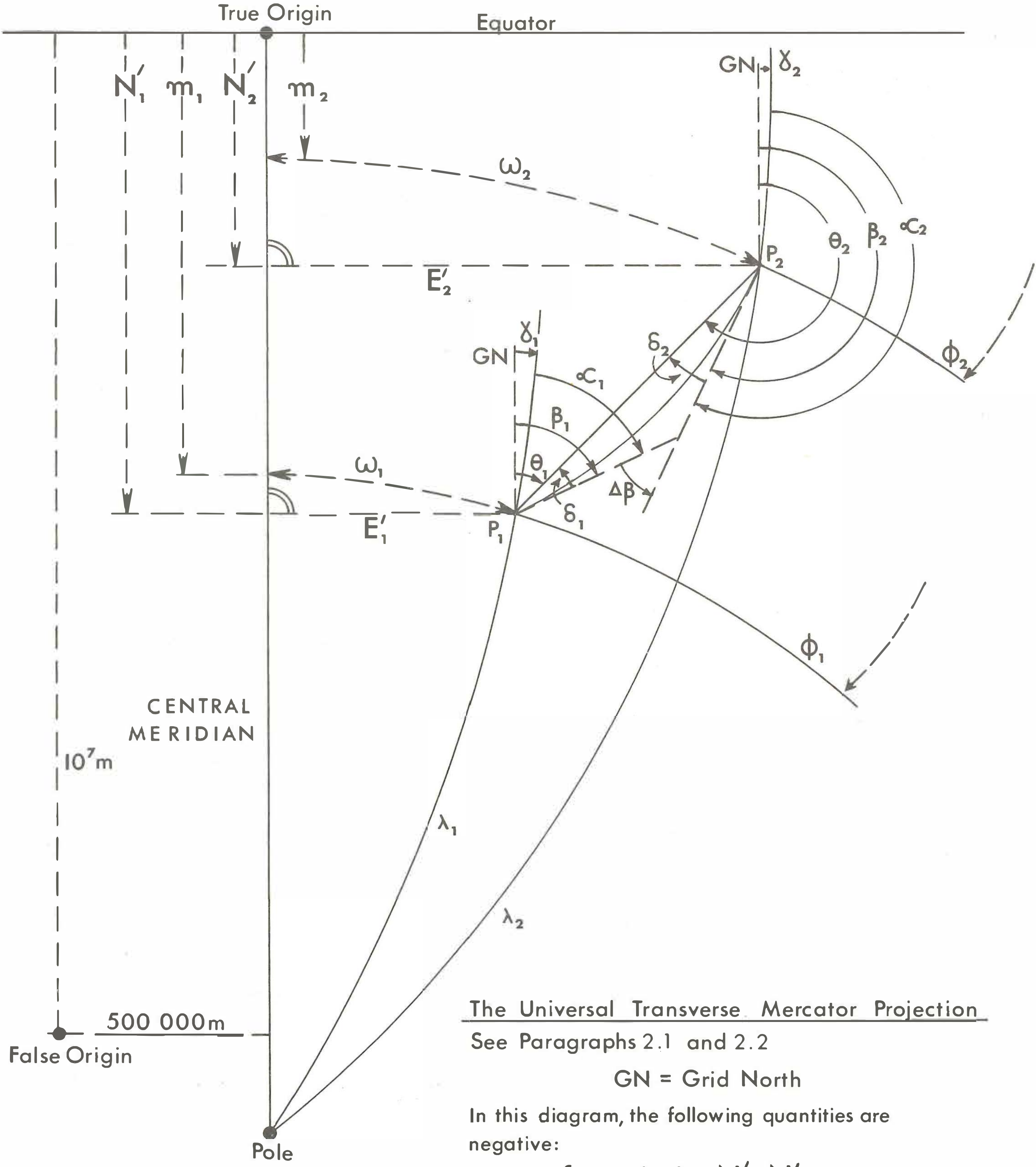
$$D_6 = q^6\ t\ (61 + 90t^2 + 45t^4 + 107e'^2\cos^2\phi - 162\,e'^2\sin^2\phi - 45\,e'^2 t^2\sin^2\phi)10^{36}/720\ \nu^6\ k_0^6\ \sin 1''$$

$$E_5 = q^5(5 + 28t^2 + 24t^4 + 6e'^2\cos^2\phi + 8e'^2\sin^2\phi)10^{30}/120\,\nu^5 k_0^5 \cos\phi\ \sin 1''$$

$$F_5 = q^5 t\ (2 + 5t^2 + 3t^4)10^{30}/15\ \nu^5\ k_0^5\ \sin 1''$$

NOTE 1. The missing Roman numerals correspond to functions absorbed in the graphs

2. Powers of ten have been introduced in order to keep the decimal point within the range of normal desk calculators.



The Universal Transverse Mercator Projection

See Paragraphs 2.1 and 2.2

GN = Grid North

In this diagram, the following quantities are negative:

$\delta_1, \Delta\beta, \phi_1, \phi_2, N_1', N_2', m_1, m_2$

All other quantities are positive.

PARAMETERS FOR ZONE TO ZONE TRANSFORMATION

ANNEX E
SHEET 1

LATITUDE	NORTHING N_z	E'_z	$\sin 2\gamma_z$	$(1-\cos 2\gamma_z)10^4$	J_1	$H_1.10^8$
0 0	10 000 000,000	333 979,761	0,000 0000	0,000	0 0	0,0000
15	9 972 329,214	333 976,597	0,000 4574	0,001	89 58	0,8261
30	9 944 658,421	333 967,106	0,000 9147	0,004	89 55	0,8260
45	9 916 987,613	333 951,287	0,001 3720	0,009	89 53	0,8260
1 0	9 889 316,781	333 929,141	0,001 8293	0,017	89 51	0,8260
15	9 861 645,919	333 900,669	0,002 2866	0,026	89 48	0,8259
30	9 833 975,018	333 865,870	0,002 7438	0,038	89 46	0,8258
45	9 806 304,072	333 824,745	0,003 2010	0,051	89 44	0,8257
2 0	9 778 633,072	333 777,296	0,003 6581	0,067	89 41	0,8256
15	9 750 962,010	333 723,523	0,004 1151	0,085	89 39	0,8254
30	9 723 290,880	333 663,427	0,004 5721	0,105	89 36	0,8253
45	9 695 619,673	333 597,008	0,005 0289	0,126	89 34	0,8251
3 0	9 667 948,382	333 524,270	0,005 4857	0,150	89 32	0,8249
15	9 640 276,999	333 445,212	0,005 9423	0,177	89 29	0,8247
30	9 612 605,516	333 359,836	0,006 3989	0,205	89 27	0,8245
45	9 584 933,927	333 268,143	0,006 8553	0,235	89 25	0,8243
4 0	9 557 262,223	333 170,136	0,007 3116	0,267	89 22	0,8240
15	9 529 590,396	333 065,816	0,007 7677	0,302	89 20	0,8238
30	9 501 918,440	332 955,185	0,008 2237	0,338	89 18	0,8235
45	9 474 246,346	332 838,245	0,008 6796	0,377	89 15	0,8232
5 0	9 446 574,107	332 714,999	0,009 1353	0,417	89 13	0,8229
15	9 418 901,716	332 585,447	0,009 5907	0,460	89 11	0,8225
30	9 391 229,164	332 449,594	0,010 0461	0,505	89 8	0,8222
45	9 363 556,445	332 307,440	0,010 5012	0,551	89 6	0,8218
6 0	9 335 883,551	332 158,990	0,010 9561	0,600	89 4	0,8215
15	9 308 210,473	332 004,245	0,011 4108	0,651	89 1	0,8211
30	9 280 537,206	331 843,208	0,011 8653	0,704	88 59	0,8207
45	9 252 863,741	331 675,883	0,012 3195	0,759	88 56	0,8202
7 0	9 225 190,071	331 502,272	0,012 7735	0,816	88 54	0,8198
15	9 197 516,188	331 322,378	0,013 2273	0,875	88 52	0,8193
30	9 169 842,085	331 136,205	0,013 6808	0,936	88 49	0,8189
45	9 142 167,755	330 943,757	0,014 1340	0,999	88 47	0,8184
8 0	9 114 493,190	330 745,036	0,014 5870	1,064	88 45	0,8179
15	9 086 818,383	330 540,046	0,015 0397	1,131	88 42	0,8173
30	9 059 143,326	330 328,791	0,015 4921	1,200	88 40	0,8168
45	9 031 468,012	330 111,276	0,015 9442	1,271	88 38	0,8163
9 0	9 003 792,434	329 887,503	0,016 3960	1,344	88 35	0,8157
15	8 976 116,584	329 657,477	0,016 8474	1,419	88 33	0,8151
30	8 948 440,455	329 421,202	0,017 2986	1,496	88 31	0,8145
45	8 920 764,040	329 178,682	0,017 7494	1,575	88 28	0,8139
10 0	8 893 087,332	328 929,923	0,018 1998	1,656	88 26	0,8133
15	8 865 410,322	328 674,928	0,018 6500	1,739	88 24	0,8126
30	8 837 733,005	328 413,702	0,019 0997	1,824	88 22	0,8119
45	8 810 055,374	328 146,250	0,019 5491	1,911	88 19	0,8113
11 0	8 782 377,419	327 872,576	0,019 9981	2,000	88 17	0,8106
15	8 754 699,136	327 592,686	0,020 4467	2,091	88 15	0,8099
30	8 727 020,516	327 306,585	0,020 8949	2,183	88 12	0,8091
45	8 699 341,552	327 014,278	0,021 3428	2,278	88 10	0,8084
12 0	8 671 662,238	326 715,771	0,021 7902	2,374	88 8	0,8076
15	8 643 982,566	326 411,068	0,022 2371	2,473	88 5	0,8069
30	8 616 302,530	326 100,175	0,022 6837	2,573	88 3	0,8061
45	8 588 622,121	325 783,099	0,023 1298	2,675	88 1	0,8053
13 0	8 560 941,335	325 459,845	0,023 5755	2,779	87 58	0,8044
15	8 533 260,162	325 130,418	0,024 0207	2,885	87 56	0,8036
30	8 505 578,598	324 794,825	0,024 4655	2,993	87 54	0,8028
45	8 477 896,634	324 453,072	0,024 9097	3,103	87 52	0,8019
14 0	8 450 214,264	324 105,165	0,025 3535	3,215	87 49	0,8010
15	8 422 531,481	323 751,110	0,025 7968	3,328	87 47	0,8001
30	8 394 848,278	323 390,915	0,026 2396	3,443	87 45	0,7992
45	8 367 164,649	323 024,585	0,026 6819	3,560	87 42	0,7983
15 0	8 339 480,586	322 652,127	0,027 1237	3,679	87 40	0,7973
15	8 311 796,084	322 273,548	0,027 5650	3,800	87 38	0,7964
30	8 284 111,135	321 888,856	0,028 0057	3,922	87 36	0,7954
45	8 256 425,733	321 498,056	0,028 4459	4,047	87 33	0,7944
16 0	8 228 739,872	321 101,157	0,028 8855	4,173	87 31	0,7934
15	8 201 053,544	320 698,165	0,029 3246	4,301	87 29	0,7924
30	8 173 366,743	320 289,087	0,029 7631	4,430	87 26	0,7913
45	8 145 679,463	319 873,932	0,030 2010	4,562	87 24	0,7903
17 0	8 117 991,697	319 452,707	0,030 6384	4,695	87 22	0,7892
15	8 090 303,439	319 025,420	0,031 0751	4,829	87 20	0,7881
30	8 062 614,683	318 592,078	0,031 5113	4,966	87 17	0,7870
45	8 034 925,422	318 152,689	0,031 9469	5,104	87 15	0,7859
18 0	8 007 235,650	317 707,262	0,032 3818	5,244	87 13	0,7848
15	7 979 545,360	317 255,804	0,032 8161	5,386	87 11	0,7837
30	7 951 854,547	316 798,324	0,033 2498	5,529	87 8	0,7825
45	7 924 163,204	316 334,830	0,033 6828	5,674	87 6	0,7813
19 0	7 896 471,325	315 865,331	0,034 1152	5,821	87 4	0,7801
15	7 868 778,904	315 389,835	0,034 5469	5,969	87 2	0,7789
30	7 841 085,934	314 908,352	0,034 9780	6,119	86 60	0,7777
45	7 813 392,411	314 420,889	0,035 4083	6,271	86 57	0,7765

PARAMETERS FOR ZONE TO ZONE TRANSFORMATION

LATITUDE	NORTHING N_z	E'_z	$\sin 2\gamma_z$	$(1-\cos 2\gamma_z)10^4$	J_1	$H_1.10^8$
20 0	7 785 698,328	313 927,455	0,035 8380	6,624	86 55	0,7752
15	7 758 003,678	313 428,061	0,036 2670	6,579	86 53	0,7740
30	7 730 308,457	312 922,715	0,036 6953	6,735	86 51	0,7727
45	7 702 612,658	312 411,426	0,037 1229	6,893	86 48	0,7714
21 0	7 674 916,275	311 894,203	0,037 5498	7,052	86 46	0,7701
15	7 647 219,302	311 371,057	0,037 9760	7,213	86 44	0,7688
30	7 619 521,735	310 841,996	0,038 4014	7,376	86 42	0,7674
45	7 591 823,567	310 307,031	0,038 8261	7,540	86 40	0,7661
22 0	7 564 124,792	309 766,171	0,039 2500	7,706	86 37	0,7647
15	7 536 425,406	309 219,426	0,039 6732	7,873	86 35	0,7633
30	7 508 725,402	308 666,806	0,040 0956	8,042	86 33	0,7619
45	7 481 024,775	308 108,322	0,040 5172	8,212	86 31	0,7605
23 0	7 453 323,519	307 543,983	0,040 9381	8,383	86 29	0,7591
15	7 425 621,629	306 973,801	0,041 3581	8,556	86 27	0,7577
30	7 397 919,100	306 397,785	0,041 7774	8,731	86 24	0,7562
45	7 370 215,927	305 815,945	0,042 1959	8,906	86 22	0,7547
24 0	7 342 512,103	305 228,294	0,042 6135	9,084	86 20	0,7533
15	7 314 807,625	304 634,841	0,043 0303	9,262	86 18	0,7518
30	7 287 102,486	304 035,598	0,043 4463	9,442	86 16	0,7502
45	7 259 396,682	303 430,575	0,043 8615	9,624	86 14	0,7487
25 0	7 231 690,208	302 819,784	0,044 2758	9,807	86 11	0,7472
15	7 203 983,058	302 203,236	0,044 6893	9,991	86 9	0,7456
30	7 176 275,228	301 580,942	0,045 1019	10,176	86 7	0,7440
45	7 148 566,712	300 952,914	0,045 5136	10,363	86 5	0,7425
26 0	7 120 857,506	300 319,163	0,045 9244	10,551	86 3	0,7409
15	7 093 147,606	299 679,700	0,046 3344	10,740	86 1	0,7393
30	7 065 437,005	299 034,539	0,046 7435	10,931	85 59	0,7376
45	7 037 725,700	298 383,690	0,047 1516	11,123	85 57	0,7360
27 0	7 010 013,686	297 727,166	0,047 5589	11,316	85 54	0,7343
15	6 982 300,958	297 064,978	0,047 9652	11,510	85 52	0,7327
30	6 954 587,512	296 397,139	0,048 3707	11,705	85 50	0,7310
45	6 926 873,343	295 723,662	0,048 7752	11,902	85 48	0,7293
28 0	6 899 158,447	295 044,558	0,049 1787	12,100	85 46	0,7276
15	6 871 442,820	294 359,840	0,049 5813	12,299	85 44	0,7258
30	6 843 726,456	293 669,521	0,049 9830	12,499	85 42	0,7241
45	6 816 009,352	292 973,614	0,050 3836	12,701	85 40	0,7224
29 0	6 788 291,504	292 272,131	0,050 7834	12,903	85 38	0,7206
15	6 760 572,907	291 565,085	0,051 1821	13,107	85 36	0,7188
30	6 732 853,558	290 852,490	0,051 5798	13,311	85 34	0,7170
45	6 705 133,452	290 134,358	0,051 9766	13,517	85 32	0,7152
30 0	6 677 412,585	289 410,703	0,052 3724	13,724	85 30	0,7134
15	6 649 690,953	288 681,538	0,052 7671	13,932	85 27	0,7116
30	6 621 968,553	287 946,877	0,053 1608	14,140	85 25	0,7097
45	6 594 245,380	287 206,733	0,053 5536	14,350	85 23	0,7078
31 0	6 566 521,432	286 461,120	0,053 9452	14,561	85 21	0,7060
15	6 538 796,703	285 710,052	0,054 3359	14,773	85 19	0,7041
30	6 511 071,191	284 953,542	0,054 7255	14,986	85 17	0,7022
45	6 483 344,892	284 191,605	0,055 1140	15,199	85 15	0,7003
32 0	6 455 617,802	283 424,255	0,055 5015	15,414	85 13	0,6983
15	6 427 889,918	282 651,506	0,055 8879	15,630	85 11	0,6964
30	6 400 161,236	281 873,372	0,056 2733	15,846	85 9	0,6944
45	6 372 431,754	281 089,867	0,056 6575	16,063	85 7	0,6925
33 0	6 344 701,467	280 301,007	0,057 0407	16,281	85 5	0,6905
15	6 316 970,373	279 506,806	0,057 4228	16,500	85 3	0,6885
30	6 289 238,469	278 707,278	0,057 8037	16,720	85 1	0,6865
45	6 261 505,750	277 902,438	0,058 1836	16,941	84 59	0,6845
34 0	6 233 772,215	277 092,302	0,058 5623	17,162	84 57	0,6824
15	6 206 037,861	276 276,884	0,058 9399	17,385	84 55	0,6804
30	6 178 302,683	275 456,199	0,059 3164	17,608	84 53	0,6783
45	6 150 566,680	274 630,263	0,059 6918	17,831	84 52	0,6763
35 0	6 122 829,849	273 799,091	0,060 0660	18,056	84 50	0,6742
15	6 095 092,187	272 962,698	0,060 4390	18,281	84 48	0,6721
30	6 067 353,691	272 121,101	0,060 8109	18,507	84 46	0,6700
45	6 039 614,358	271 274,314	0,061 1816	18,733	84 44	0,6678
36 0	6 011 874,187	270 422,353	0,061 5511	18,961	84 42	0,6657
15	5 984 133,175	269 565,234	0,061 9195	19,189	84 40	0,6636
30	5 956 391,318	268 702,974	0,062 2866	19,417	84 38	0,6614
45	5 928 648,615	267 835,587	0,062 6526	19,646	86 36	0,6592
37 0	5 900 905,064	266 963,091	0,063 0174	19,876	84 34	0,6570
15	5 873 160,663	266 085,501	0,063 3809	20,106	84 32	0,6548
30	5 845 415,408	265 202,834	0,063 7433	20,337	84 30	0,6526
45	5 817 669,298	264 315,106	0,064 1044	20,568	84 29	0,6504
38 0	5 789 922,332	263 422,334	0,064 4643	20,800	84 27	0,6482
15	5 762 174,506	262 524,534	0,064 8229	21,032	84 25	0,6459
30	5 734 425,820	261 621,724	0,065 1803	21,265	84 23	0,6437
45	5 706 676,270	260 713,919	0,065 5364	21,498	84 21	0,6414
39 0	5 678 925,857	259 801,137	0,065 8913	21,732	84 19	0,6391
15	5 651 174,577	258 883,395	0,066 2450	21,966	84 17	0,6368
30	5 623 422,429	257 960,710	0,066 5973	22,201	84 16	0,6345
45	5 595 669,411	257 033,100	0,066 9484	22,436	84 14	0,6322

PARAMETERS FOR ZONE TO ZONE TRANSFORMATION

LATITUDE	NORTHING N_z	E'_z	$\sin 2\gamma_z$	$(1-\cos 2\gamma_z)10^4$	J_1	$H_1.10^8$
40 0	5 567 915,523	256 100,580	0,067 2982	22,671	84 12	0,6299
15	5 540 160,762	255 163,170	0,067 6467	22,907	84 10	0,6275
30	5 512 405,128	254 220,885	0,067 9939	23,143	84 8	0,6252
45	5 484 648,618	253 273,745	0,068 3398	23,379	84 7	0,6228
41 0	5 456 891,232	252 321,767	0,068 6844	23,616	84 5	0,6204
15	5 429 132,968	251 364,968	0,069 0277	23,853	84 3	0,6180
30	5 401 373,826	250 403,366	0,069 3696	24,090	84 1	0,6156
45	5 373 613,803	249 436,979	0,069 7103	24,327	83 59	0,6132
42 0	5 345 852,901	248 465,826	0,070 0496	24,565	83 58	0,6108
15	5 318 091,116	247 489,924	0,070 3875	24,803	83 56	0,6084
30	5 290 328,449	246 509,292	0,070 7241	25,041	83 54	0,6059
45	5 262 564,899	245 523,948	0,071 0593	25,279	83 52	0,6035
43 0	5 234 800,464	244 533,911	0,071 3932	25,518	83 51	0,6010
15	5 207 035,145	243 539,198	0,071 7257	25,756	83 49	0,5985
30	5 179 268,941	242 539,829	0,072 0569	25,995	83 47	0,5960
45	5 151 501,851	241 535,823	0,072 3866	26,234	83 45	0,5935
44 0	5 123 733,875	240 527,198	0,072 7150	26,472	83 44	0,5910
15	5 095 965,012	239 513,973	0,073 0420	26,711	83 42	0,5885
30	5 068 195,262	238 496,167	0,073 3676	26,950	83 40	0,5859
45	5 040 424,626	237 473,799	0,073 6917	27,189	83 39	0,5834
45 0	5 012 653,101	236 446,888	0,074 0145	27,428	83 37	0,5808
15	4 984 880,690	235 415,454	0,074 3358	27,667	83 35	0,5783
30	4 957 107,391	234 379,516	0,074 6558	27,906	83 34	0,5757
45	4 929 333,204	233 339,094	0,074 9743	28,145	83 32	0,5731
46 0	4 901 558,130	232 294,206	0,075 2913	28,384	83 30	0,5705
15	4 873 782,169	231 244,873	0,075 6069	28,623	83 29	0,5679
30	4 846 005,321	230 191,114	0,075 9211	28,862	83 27	0,5653
45	4 818 227,586	229 132,949	0,076 2338	29,100	83 25	0,5626
47 0	4 790 448,965	228 070,397	0,076 5451	29,339	83 24	0,5600
15	4 762 669,458	227 003,480	0,076 8549	29,577	83 22	0,5573
30	4 734 889,065	225 932,217	0,077 1632	29,815	83 21	0,5547
45	4 707 107,788	224 856,627	0,077 4701	30,053	83 19	0,5520
48 0	4 679 325,626	223 776,732	0,077 7755	30,291	83 17	0,5493
15	4 651 542,581	222 692,552	0,078 0793	30,529	83 16	0,5466
30	4 623 758,654	221 604,107	0,078 3817	30,766	83 14	0,5439
45	4 595 973,844	220 511,417	0,078 6826	31,003	83 13	0,5412
49 0	4 568 188,153	219 414,503	0,078 9820	31,240	83 11	0,5385
15	4 540 401,582	218 313,386	0,079 2799	31,476	83 9	0,5357
30	4 512 614,133	217 208,086	0,079 5763	31,712	83 8	0,5330
45	4 484 825,805	216 098,624	0,079 8711	31,948	83 6	0,5302
50 0	4 457 036,601	214 985,022	0,080 1644	32,183	83 5	0,5275
15	4 429 246,521	213 867,300	0,080 4562	32,419	83 3	0,5247
30	4 401 455,567	212 745,479	0,080 7465	32,653	83 2	0,5219
45	4 373 663,740	211 619,581	0,081 0352	32,888	83 0	0,5191
51 0	4 345 871,042	210 489,626	0,081 3223	33,121	82 59	0,5163
15	4 318 077,474	209 355,637	0,081 6079	33,355	82 57	0,5135
30	4 290 283,037	208 217,633	0,081 8920	33,588	82 56	0,5107
45	4 262 487,734	207 075,638	0,082 1745	33,820	82 54	0,5079
52 0	4 234 691,566	205 929,672	0,082 4554	34,052	82 53	0,5050
15	4 206 894,535	204 779,757	0,082 7347	34,284	82 51	0,5022
30	4 179 096,642	203 625,915	0,083 0125	34,515	82 50	0,4993
45	4 151 297,890	202 468,167	0,083 2887	34,745	82 48	0,4964
53 0	4 123 498,281	201 306,536	0,083 5632	34,975	82 47	0,4936
15	4 095 697,816	200 141,044	0,083 8362	35,205	82 46	0,4907
30	4 067 896,498	198 971,712	0,084 1076	35,433	82 44	0,4878
45	4 040 094,328	197 798,562	0,084 3774	35,661	82 43	0,4849
54 0	4 012 291,310	196 621,618	0,084 6456	35,889	82 41	0,4820
15	3 984 487,445	195 440,900	0,084 9122	36,116	82 40	0,4790
30	3 956 682,736	194 256,432	0,085 1771	36,342	82 39	0,4761
45	3 928 877,185	193 068,236	0,085 4404	36,567	82 37	0,4732
55 0	3 901 070,794	191 876,335	0,085 7021	36,792	82 36	0,4702
15	3 873 263,567	190 680,750	0,085 9622	37,016	82 34	0,4673
30	3 845 455,506	189 481,505	0,086 2206	37,239	82 33	0,4643
45	3 817 646,613	188 278,622	0,086 4773	37,462	82 32	0,4613
56 0	3 789 836,891	187 072,124	0,086 7325	37,684	82 30	0,4584

LATITUDE FUNCTIONS FOR POINT TO POINT COMPUTATION

LATITUDE	NORTHING	$10^{15}/2r^2$	Diff 1′	$10^{15}/6r^2$	Diff 1′	$10^{10}/2r^2\sin1''$	Diff 1′	$10^{10}/6r^2\sin1''$	Diff 1′
00 00	10 000 000	12.383 51	0.0	4.127 84	0.0	25.542 83	0.0	8.514 28	0.0
10	9 982 000	12.383 51	0.0	4.127 84	0.0	25.542 83	-0.1	8.514 28	-0.1
20	9 963 000	12.383 51	-0.1	4.127 84	-0.1	25.542 82	-0.2	8.514 27	0.0
30	9 945 000	12.383 50	-0.1	4.127 83	0.0	25.542 80	-0.2	8.514 27	-0.1
40	9 926 000	12.383 49	-0.1	4.127 83	0.0	25.542 78	-0.2	8.514 26	-0.1
50	9 908 000	12.383 48	-0.2	4.127 83	-0.1	25.542 76	-0.3	8.514 25	-0.1
01 00	9 889 000	12.383 46	-0.2	4.127 82	-0.1	25.542 73	-0.4	8.514 24	-0.1
10	9 871 000	12.383 44	-0.2	4.127 81	0.0	25.542 69	-0.5	8.514 23	-0.2
20	9 853 000	12.383 42	-0.2	4.127 81	-0.1	25.542 64	-0.5	8.514 21	-0.1
30	9 834 000	12.383 40	-0.3	4.127 80	-0.1	25.542 59	-0.5	8.514 20	-0.2
40	9 816 000	12.383 37	-0.3	4.127 79	-0.1	25.542 54	-0.6	8.514 18	-0.2
50	9 797 000	12.383 34	-0.3	4.127 78	-0.1	25.542 48	-0.7	8.514 16	-0.2
02 00	9 779 000	12.383 31	-0.3	4.127 77	-0.1	25.542 41	-0.7	8.514 14	-0.3
10	9 760 000	12.383 28	-0.4	4.127 76	-0.1	25.542 34	-0.8	8.514 11	-0.2
20	9 742 000	12.383 24	-0.4	4.127 75	-0.2	25.542 26	-0.8	8.514 09	-0.3
30	9 724 000	12.383 20	-0.5	4.127 73	-0.1	25.542 18	-0.9	8.514 06	-0.3
40	9 705 000	12.383 15	-0.4	4.127 72	-0.2	25.542 09	-1.0	8.514 03	-0.3
50	9 687 000	12.383 11	-0.5	4.127 70	-0.1	25.541 99	-1.0	8.514 00	-0.4
03 00	9 668 000	12.383 06	-0.5	4.127 69	-0.2	25.541 89	-1.0	8.513 96	-0.3
10	9 650 000	12.383 01	-0.6	4.127 67	-0.2	25.541 79	-1.2	8.513 93	-0.4
20	9 631 000	12.382 95	-0.5	4.127 65	-0.2	25.541 67	-1.2	8.513 89	-0.4
30	9 613 000	12.382 90	-0.6	4.127 63	-0.2	25.541 55	-1.2	8.513 85	-0.4
40	9 595 000	12.382 84	-0.7	4.127 61	-0.2	25.541 43	-1.3	8.513 81	-0.4
50	9 576 000	12.382 77	-0.6	4.127 59	-0.2	25.541 30	-1.3	8.513 77	-0.5
04 00	9 558 000	12.382 71	-0.7	4.127 57	-0.2	25.541 17	-1.5	8.513 72	-0.5
10	9 539 000	12.382 64	-0.7	4.127 55	-0.3	25.541 02	-1.4	8.513 67	-0.4
20	9 521 000	12.382 57	-0.8	4.127 52	-0.2	25.540 88	-1.6	8.513 63	-0.6
30	9 502 000	12.382 49	-0.7	4.127 50	-0.3	25.540 72	-1.5	8.513 57	-0.5
40	9 484 000	12.382 42	-0.8	4.127 47	-0.2	25.540 57	-1.7	8.513 52	-0.5
50	9 466 000	12.382 34	-0.9	4.127 45	-0.3	25.540 40	-1.7	8.513 47	-0.6
05 00	9 447 000	12.382 25	-0.8	4.127 42	-0.3	25.540 23	-1.7	8.513 41	-0.6
10	9 429 000	12.382 17	-0.9	4.127 39	-0.3	25.540 06	-1.9	8.513 35	-0.6
20	9 410 000	12.382 08	-0.9	4.127 36	-0.3	25.539 87	-1.8	8.513 29	-0.6
30	9 392 000	12.381 99	-0.9	4.127 33	-0.3	25.539 69	-1.9	8.513 23	-0.6
40	9 373 000	12.381 90	-1.0	4.127 30	-0.3	25.539 50	-2.0	8.513 17	-0.7
50	9 355 000	12.381 80	-1.0	4.127 27	-0.4	25.539 30	-2.1	8.513 10	-0.7
06 00	9 337 000	12.381 70	-1.0	4.127 23	-0.3	25.539 09	-2.1	8.513 03	-0.7
10	9 318 000	12.381 60	-1.0	4.127 20	-0.3	25.538 88	-2.1	8.512 96	-0.7
20	9 300 000	12.381 50	-1.1	4.127 17	-0.4	25.538 67	-2.2	8.512 89	-0.7
30	9 281 000	12.381 39	-1.1	4.127 13	-0.4	25.538 45	-2.3	8.512 82	-0.8
40	9 263 000	12.381 28	-1.1	4.127 09	-0.3	25.538 22	-2.3	8.512 74	-0.8
50	9 244 000	12.381 17	-1.2	4.127 06	-0.4	25.537 99	-2.4	8.512 66	-0.8
07 00	9 226 000	12.381 05	-1.2	4.127 02	-0.4	25.537 75	-2.4	8.512 58	-0.8
10	9 208 000	12.380 93	-1.2	4.126 98	-0.4	25.537 51	-2.5	8.512 50	-0.8
20	9 189 000	12.380 81	-1.2	4.126 94	-0.4	25.537 26	-2.6	8.512 42	-0.9
30	9 171 000	12.380 69	-1.3	4.126 90	-0.5	25.537 00	-2.6	8.512 33	-0.8
40	9 152 000	12.380 56	-1.3	4.126 85	-0.4	25.536 74	-2.6	8.512 25	-0.9
50	9 134 000	12.380 43	-1.3	4.126 81	-0.4	25.536 48	-2.7	8.512 16	-0.9
08 00	9 115 000	12.380 30	-1.3	4.126 77	-0.5	25.536 21	-2.8	8.512 07	-0.9
10	9 097 000	12.380 17	-1.4	4.126 72	-0.4	25.535 93	-2.8	8.511 98	-1.0
20	9 079 000	12.380 03	-1.4	4.126 68	-0.5	25.535 65	-2.9	8.511 88	-0.9
30	9 060 000	12.379 89	-1.4	4.126 63	-0.5	25.535 36	-3.0	8.511 79	-1.0
40	9 042 000	12.379 75	-1.5	4.126 58	-0.5	25.535 06	-2.9	8.511 69	-1.0
50	9 023 000	12.379 60	-1.4	4.126 53	-0.4	25.534 77	-3.1	8.511 59	-1.0
09 00	9 005 000	12.379 46	-1.5	4.126 49	-0.5	25.534 46	-3.1	8.511 49	-1.1
10	8 986 000	12.379 31	-1.6	4.126 44	-0.6	25.534 15	-3.1	8.511 38	-1.0
20	8 968 000	12.379 15	-1.5	4.126 38	-0.5	25.533 84	-3.3	8.511 28	-1.1
30	8 950 000	12.379 00	-1.6	4.126 33	-0.5	25.533 51	-3.2	8.511 17	-1.1
40	8 931 000	12.378 84	-1.6	4.126 28	-0.5	25.533 19	-3.3	8.511 06	-1.1
50	8 913 000	12.378 68	-1.7	4.126 23	-0.6	25.532 86	-3.4	8.510 95	-1.1
10 00	8 894 000	12.378 51	-1.6	4.126 17	-0.5	25.532 52	-3.4	8.510 84	-1.1
10	8 876 000	12.378 35	-1.7	4.126 12	-0.6	25.532 18	-3.5	8.510 73	-1.2
20	8 857 000	12.378 18	-1.7	4.126 06	-0.6	25.531 83	-3.6	8.510 61	-1.2
30	8 839 000	12.378 01	-1.8	4.126 00	-0.6	25.531 47	-3.6	8.510 49	-1.2
40	8 820 000	12.377 83	-1.7	4.125 94	-0.5	25.531 11	-3.6	8.510 37	-1.2
50	8 802 000	12.377 66	-1.8	4.125 89	-0.6	25.530 75	-3.7	8.510 25	-1.2
11 00	8 784 000	12.377 48	-1.8	4.125 83	-0.6	25.530 38	-3.8	8.510 13	-1.3
10	8 765 000	12.377 30	-1.9	4.125 77	-0.7	25.530 00	-3.8	8.510 00	-1.3
20	8 747 000	12.377 11	-1.9	4.125 70	-0.6	25.529 62	-3.8	8.509 87	-1.2
30	8 728 000	12.376 92	-1.9	4.125 64	-0.6	25.529 24	-3.9	8.509 75	-1.3
40	8 710 000	12.376 73	-1.9	4.125 58	-0.7	25.528 85	-4.0	8.509 62	-1.4
50	8 691 000	12.376 54	-1.9	4.125 51	-0.6	25.528 45	-4.0	8.509 48	-1.3
12 00	8 673 000	12.376 35	-2.0	4.125 45	-0.7	25.528 05	-4.1	8.509 35	-1.4
10	8 655 000	12.376 15	-2.0	4.125 38	-0.6	25.527 64	-4.1	8.509 21	-1.3
20	8 636 000	12.375 95	-2.0	4.125 32	-0.7	25.527 23	-4.2	8.509 08	-1.4
30	8 618 000	12.375 75	-2.1	4.125 25	-0.7	25.526 81	-4.2	8.508 94	-1.4
40	8 599 000	12.375 54	-2.1	4.125 18	-0.7	25.526 39	-4.3	8.508 80	-1.5
50	8 581 000	12.375 33	-2.1	4.125 11	-0.7	25.525 96	-4.3	8.508 65	-1.4
13 00	8 562 000	12.375 12	-2.1	4.125 04	-0.7	25.525 53	-4.4	8.508 51	-1.5
10	8 544 000	12.374 91	-2.1	4.124 97	-0.7	25.525 09	-4.5	8.508 36	-1.5
20	8 526 000	12.374 70	-2.2	4.124 90	-0.7	25.524 64	-4.4	8.508 21	-1.4
30	8 507 000	12.374 48	-2.2	4.124 83	-0.8	25.524 20	-4.6	8.508 07	-1.6
40	8 489 000	12.374 26	-2.2	4.124 75	-0.7	25.523 74	-4.6	8.507 91	-1.5
50	8 470 000	12.374 04	-2.3	4.124 68	-0.8	25.523 28	-4.6	8.507 76	-1.5
14 00	8 452 000	12.373 81	-2.3	4.124 60	-0.7	25.522 82	-4.7	8.507 61	-1.6
10	8 433 000	12.373 58	-2.3	4.124 53	-0.8	25.522 35	-4.8	8.507 45	-1.6
20	8 415 000	12.373 35	-2.3	4.124 45	-0.8	25.521 87	-4.8	8.507 29	-1.6
30	8 396 000	12.373 12	-2.3	4.124 37	-0.7	25.521 39	-4.8	8.507 13	-1.6
40	8 378 000	12.372 89	-2.4	4.124 30	-0.8	25.520 91	-4.9	8.506 97	-1.6
50	8 360 000	12.372 65	-2.4	4.124 22	-0.8	25.520 42	-4.9	8.506 81	-1.7

NOTE: Northings are approximate values rounded to the nearest 1,000 metres

LATITUDE FUNCTIONS FOR POINT TO POINT COMPUTATION

LATITUDE	NORTHING	$10^{15}/2r^2$	Diff 1′	$10^{15}/6r^2$	Diff 1′	$10^{10}/2r^2\sin1''$	Diff 1′	$10^{10}/6r^2\sin1''$	Diff 1′
15 00	8 341 000	12.372 41	-2.4	4.124 14	-0.8	25.519 93	-5.0	8.506 64	-1.6
10	8 323 000	12.372 17	-2.5	4.124 06	-0.9	25.519 43	-5.1	8.506 48	-1.7
20	8 304 000	12.371 92	-2.5	4.123 97	-0.8	25.518 92	-5.1	8.506 31	-1.7
30	8 286 000	12.371 67	-2.4	4.123 89	-0.8	25.518 41	-5.1	8.506 14	-1.7
40	8 267 000	12.371 43	-2.6	4.123 81	-0.9	25.517 90	-5.2	8.505 97	-1.8
50	8 249 000	12.371 17	-2.5	4.123 72	-0.8	25.517 38	-5.3	8.505 79	-1.7
16 00	8 230 000	12.370 92	-2.6	4.123 64	-0.9	25.516 85	-5.3	8.505 62	-1.8
10	8 212 000	12.370 66	-2.6	4.123 55	-0.8	25.516 32	-5.3	8.505 44	-1.8
20	8 194 000	12.370 40	-2.6	4.123 47	-0.9	25.515 79	-5.4	8.505 26	-1.8
30	8 175 000	12.370 14	-2.6	4.123 38	-0.9	25.515 25	-5.4	8.505 08	-1.8
40	8 157 000	12.369 88	-2.7	4.123 29	-0.9	25.514 71	-5.5	8.504 90	-1.8
50	8 138 000	12.369 61	-2.7	4.123 20	-0.9	25.514 16	-5.6	8.504 72	-1.9
17 00	8 120 000	12.369 34	-2.7	4.123 11	-0.9	25.513 60	-5.5	8.504 53	-1.8
10	8 101 000	12.369 07	-2.7	4.123 02	-0.9	25.513 05	-5.7	8.504 35	-1.9
20	8 083 000	12.368 80	-2.7	4.122 93	-0.9	25.512 48	-5.7	8.504 16	-1.9
30	8 064 000	12.368 53	-2.8	4.122 84	-0.9	25.511 91	-5.7	8.503 97	-1.9
40	8 046 000	12.368 25	-2.8	4.122 75	-0.9	25.511 34	-5.8	8.503 78	-1.9
50	8 028 000	12.367 97	-2.8	4.122 66	-1.0	25.510 76	-5.8	8.503 59	-2.0
18 00	8 009 000	12.367 69	-2.9	4.122 56	-0.9	25.510 18	-5.8	8.503 39	-1.9
10	7 991 000	12.367 40	-2.9	4.122 47	-1.0	25.509 60	-6.0	8.503 20	-2.0
20	7 972 000	12.367 11	-2.8	4.122 37	-0.9	25.509 00	-5.9	8.503 00	-2.0
30	7 954 000	12.366 83	-3.0	4.122 28	-1.0	25.508 41	-6.0	8.502 80	-2.0
40	7 935 000	12.366 53	-2.9	4.122 18	-1.0	25.507 81	-6.1	8.502 60	-2.0
50	7 917 000	12.366 24	-3.0	4.122 08	-1.0	25.507 20	-6.1	8.502 40	-2.0
19 00	7 898 000	12.365 94	-2.9	4.121 98	-1.0	25.506 59	-6.1	8.502 20	-2.1
10	7 880 000	12.365 65	-3.0	4.121 88	-1.0	25.505 98	-6.2	8.501 99	-2.0
20	7 862 000	12.365 35	-3.1	4.121 78	-1.0	25.505 36	-6.2	8.501 79	-2.1
30	7 843 000	12.365 04	-3.0	4.121 68	-1.0	25.504 74	-6.3	8.501 58	-2.1
40	7 825 000	12.364 74	-3.1	4.121 58	-1.0	25.504 11	-6.3	8.501 37	-2.1
50	7 806 000	12.364 43	-3.0	4.121 48	-1.0	25.503 48	-6.4	8.501 16	-2.1
20 00	7 788 000	12.364 13	-3.2	4.121 38	-1.1	25.502 84	-6.4	8.500 95	-2.2
10	7 769 000	12.363 81	-3.1	4.121 27	-1.0	25.502 20	-6.5	8.500 73	-2.1
20	7 751 000	12.363 50	-3.1	4.121 17	-1.1	25.501 55	-6.5	8.500 52	-2.2
30	7 732 000	12.363 19	-3.2	4.121 06	-1.0	25.500 90	-6.5	8.500 30	-2.2
40	7 714 000	12.362 87	-3.2	4.120 96	-1.1	25.500 25	-6.6	8.500 08	-2.2
50	7 696 000	12.362 55	-3.2	4.120 85	-1.1	25.499 59	-6.6	8.499 86	-2.2
21 00	7 677 000	12.362 23	-3.2	4.120 74	-1.1	25.498 93	-6.7	8.499 64	-2.2
10	7 659 000	12.361 91	-3.3	4.120 63	-1.0	25.498 26	-6.7	8.499 42	-2.2
20	7 640 000	12.361 58	-3.3	4.120 53	-1.1	25.497 59	-6.8	8.499 20	-2.3
30	7 622 000	12.361 25	-3.3	4.120 42	-1.1	25.496 91	-6.8	8.498 97	-2.3
40	7 603 000	12.360 92	-3.3	4.120 31	-1.1	25.496 23	-6.8	8.498 74	-2.2
50	7 585 000	12.360 59	-3.3	4.120 20	-1.1	25.495 55	-6.9	8.498 52	-2.3
22 00	7 566 000	12.360 26	-3.4	4.120 09	-1.2	25.494 86	-6.9	8.498 29	-2.3
10	7 548 000	12.359 92	-3.4	4.119 97	-1.1	25.494 17	-7.0	8.498 06	-2.4
20	7 529 000	12.359 58	-3.4	4.119 86	-1.1	25.493 47	-7.0	8.497 82	-2.3
30	7 511 000	12.359 24	-3.4	4.119 75	-1.2	25.492 77	-7.0	8.497 59	-2.3
40	7 493 000	12.358 90	-3.4	4.119 63	-1.1	25.492 07	-7.1	8.497 36	-2.4
50	7 474 000	12.358 56	-3.5	4.119 52	-1.2	25.491 36	-7.2	8.497 12	-2.4
23 00	7 456 000	12.358 21	-3.4	4.119 40	-1.1	25.490 64	-7.1	8.496 88	-2.4
10	7 437 000	12.357 87	-3.5	4.119 29	-1.2	25.489 93	-7.2	8.496 64	-2.4
20	7 419 000	12.357 52	-3.6	4.119 17	-1.2	25.489 21	-7.3	8.496 40	-2.4
30	7 400 000	12.357 16	-3.5	4.119 05	-1.1	25.488 48	-7.3	8.496 16	-2.4
40	7 382 000	12.356 81	-3.5	4.118 94	-1.2	25.487 75	-7.3	8.495 92	-2.5
50	7 363 000	12.356 46	-3.6	4.118 82	-1.2	25.487 02	-7.4	8.495 67	-2.4
24 00	7 345 000	12.356 10	-3.6	4.118 70	-1.2	25.486 28	-7.4	8.495 43	-2.5
10	7 326 000	12.355 74	-3.6	4.118 58	-1.2	25.485 54	-7.4	8.495 18	-2.5
20	7 308 000	12.355 38	-3.6	4.118 46	-1.2	25.484 80	-7.5	8.494 93	-2.5
30	7 290 000	12.355 02	-3.7	4.118 34	-1.2	25.484 05	-7.5	8.494 68	-2.5
40	7 271 000	12.354 65	-3.6	4.118 22	-1.2	25.483 30	-7.6	8.494 43	-2.5
50	7 253 000	12.354 29	-3.7	4.118 10	-1.3	25.482 54	-7.6	8.494 18	-2.5
25 00	7 234 000	12.353 92	-3.7	4.117 97	-1.2	25.481 78	-7.6	8.493 93	-2.6
10	7 216 000	12.353 55	-3.7	4.117 85	-1.2	25.481 02	-7.7	8.493 67	-2.5
20	7 197 000	12.353 18	-3.8	4.117 73	-1.3	25.480 25	-7.7	8.493 42	-2.6
30	7 179 000	12.352 80	-3.7	4.117 60	-1.2	25.479 48	-7.7	8.493 16	-2.6
40	7 160 000	12.352 43	-3.8	4.117 48	-1.3	25.478 71	-7.8	8.492 90	-2.6
50	7 142 000	12.352 05	-3.8	4.117 35	-1.3	25.477 93	-7.8	8.492 64	-2.6
26 00	7 123 000	12.351 67	-3.8	4.117 22	-1.2	25.477 15	-7.8	8.492 38	-2.6
10	7 105 000	12.351 29	-3.8	4.117 10	-1.3	25.476 37	-7.9	8.492 12	-2.6
20	7 087 000	12.350 91	-3.9	4.116 97	-1.3	25.475 58	-7.9	8.491 86	-2.6
30	7 068 000	12.350 52	-3.8	4.116 84	-1.3	25.474 79	-8.0	8.491 60	-2.7
40	7 050 000	12.350 14	-3.9	4.116 71	-1.3	25.473 99	-8.0	8.491 33	-2.7
50	7 031 000	12.349 75	-3.9	4.116 58	-1.3	25.473 19	-8.0	8.491 06	-2.6
27 00	7 013 000	12.349 36	-3.9	4.116 45	-1.3	25.472 39	-8.1	8.490 80	-2.7
10	6 994 000	12.348 97	-3.9	4.116 32	-1.3	25.471 58	-8.0	8.490 53	-2.7
20	6 976 000	12.348 58	-3.9	4.116 19	-1.3	25.470 78	-8.2	8.490 26	-2.7
30	6 957 000	12.348 19	-4.0	4.116 06	-1.3	25.469 96	-8.1	8.489 99	-2.7
40	6 939 000	12.347 79	-4.0	4.115 93	-1.3	25.469 15	-8.2	8.489 72	-2.8
50	6 920 000	12.347 39	-3.9	4.115 80	-1.3	25.468 33	-8.2	8.489 44	-2.7
28 00	6 902 000	12.347 00	-4.0	4.115 67	-1.4	25.467 51	-8.3	8.489 17	-2.8
10	6 883 000	12.346 60	-4.1	4.115 53	-1.3	25.466 68	-8.3	8.488 89	-2.7
20	6 865 000	12.346 19	-4.0	4.115 40	-1.4	25.465 85	-8.3	8.488 62	-2.8
30	6 846 000	12.345 79	-4.0	4.115 26	-1.3	25.465 02	-8.3	8.488 34	-2.8
40	6 828 000	12.345 39	-4.1	4.115 13	-1.4	25.464 19	-8.4	8.488 06	-2.8
50	6 810 000	12.344 98	-4.1	4.114 99	-1.3	25.463 35	-8.4	8.487 78	-2.8
29 00	6 791 000	12.344 57	-4.1	4.114 86	-1.4	25.462 51	-8.4	8.487 50	-2.8
10	6 773 000	12.344 16	-4.1	4.114 72	-1.4	25.461 67	-8.5	8.487 22	-2.8
20	6 754 000	12.343 75	-4.1	4.114 58	-1.3	25.460 82	-8.5	8.486 94	-2.8
30	6 736 000	12.343 34	-4.1	4.114 45	-1.4	25.459 97	-8.5	8.486 66	-2.9
40	6 717 000	12.342 93	-4.2	4.114 31	-1.4	25.459 12	-8.6	8.486 37	-2.8
50	6 699 000	12.342 51	-4.1	4.114 17	-1.4	25.458 26	-8.6	8.486 09	-2.9

NOTE: Northings are approximate values rounded to the nearest 1,000 metres

LATITUDE FUNCTIONS FOR POINT TO POINT COMPUTATION

ANNEX F
SHEET 3

LATITUDE	NORTHING	$10^{15}/2r^2$	Diff 1′	$10^{15}/6r^2$	Diff 1′	$10^{10}/2r^2\sin 1''$	Diff 1′	$10^{10}/6r^2\sin 1''$	Diff 1′
30 00	6 680 000	12.342 10	-4.2	4.114 03	-1.4	25.457 40	-8.6	8.485 80	-2.9
10	6 662 000	12.341 68	-4.2	4.113 89	-1.4	25.456 54	-8.6	8.485 51	-2.8
20	6 643 000	12.341 26	-4.2	4.113 75	-1.4	25.455 68	-8.7	8.485 23	-2.9
30	6 625 000	12.340 84	-4.2	4.113 61	-1.4	25.454 81	-8.7	8.484 94	-2.9
40	6 606 000	12.340 42	-4.3	4.113 47	-1.4	25.453 94	-8.7	8.484 65	-2.9
50	6 588 000	12.339 99	-4.2	4.113 33	-1.4	25.453 07	-8.8	8.484 36	-3.0
31 00	6 569 000	12.339 57	-4.3	4.113 19	-1.4	25.452 19	-8.8	8.484 06	-2.9
10	6 551 000	12.339 14	-4.2	4.113 05	-1.4	25.451 31	-8.8	8.483 77	-2.9
20	6 532 000	12.338 72	-4.3	4.112 91	-1.5	25.450 43	-8.8	8.483 48	-3.0
30	6 514 000	12.338 29	-4.3	4.112 76	-1.4	25.449 55	-8.9	8.483 18	-2.9
40	6 496 000	12.337 86	-4.3	4.112 62	-1.4	25.448 66	-8.9	8.482 89	-3.0
50	6 477 000	12.337 43	-4.3	4.112 48	-1.5	25.447 77	-8.9	8.482 59	-3.0
32 00	6 459 000	12.337 00	-4.4	4.112 33	-1.4	25.446 88	-8.9	8.482 29	-2.9
10	6 440 000	12.336 56	-4.3	4.112 19	-1.5	25.445 99	-9.0	8.482 00	-3.0
20	6 422 000	12.336 13	-4.4	4.112 04	-1.4	25.445 09	-9.0	8.481 70	-3.0
30	6 403 000	12.335 69	-4.3	4.111 90	-1.5	25.444 19	-9.0	8.481 40	-3.0
40	6 385 000	12.335 26	-4.4	4.111 75	-1.4	25.443 29	-9.0	8.481 10	-3.0
50	6 366 000	12.334 82	-4.4	4.111 61	-1.5	25.442 39	-9.1	8.480 80	-3.1
33 00	6 348 000	12.334 38	-4.4	4.111 46	-1.5	25.441 48	-9.0	8.480 49	-3.0
10	6 329 000	12.333 94	-4.4	4.111 31	-1.4	25.440 58	-9.2	8.480 19	-3.0
20	6 311 000	12.333 50	-4.4	4.111 17	-1.5	25.439 66	-9.1	8.479 89	-3.1
30	6 292 000	12.333 06	-4.5	4.111 02	-1.5	25.438 75	-9.1	8.479 58	-3.0
40	6 274 000	12.332 61	-4.4	4.110 87	-1.5	25.437 84	-9.2	8.479 28	-3.1
50	6 255 000	12.332 17	-4.5	4.110 72	-1.5	25.436 92	-9.2	8.478 97	-3.0
34 00	6 237 000	12.331 72	-4.5	4.110 57	-1.5	25.436 00	-9.2	8.478 67	-3.1
10	6 218 000	12.331 27	-4.4	4.110 42	-1.4	25.435 08	-9.2	8.478 36	-3.1
20	6 200 000	12.330 83	-4.5	4.110 28	-1.5	25.434 16	-9.3	8.478 05	-3.1
30	6 181 000	12.330 38	-4.5	4.110 13	-1.5	25.433 23	-9.3	8.477 74	-3.1
40	6 163 000	12.329 93	-4.5	4.109 98	-1.5	25.432 30	-9.3	8.477 43	-3.1
50	6 144 000	12.329 48	-4.5	4.109 83	-1.5	25.431 37	-9.3	8.477 12	-3.1
35 00	6 126 000	12.329 03	-4.6	4.109 68	-1.6	25.430 44	-9.3	8.476 81	-3.1
10	6 107 000	12.328 57	-4.5	4.109 52	-1.5	25.429 51	-9.4	8.476 50	-3.1
20	6 089 000	12.328 12	-4.6	4.109 37	-1.5	25.428 57	-9.4	8.476 19	-3.1
30	6 070 000	12.327 66	-4.5	4.109 22	-1.5	25.427 63	-9.4	8.475 88	-3.2
40	6 052 000	12.327 21	-4.6	4.109 07	-1.5	25.426 69	-9.4	8.475 56	-3.1
50	6 033 000	12.326 75	-4.5	4.108 92	-1.5	25.425 75	-9.4	8.475 25	-3.1
36 00	6 015 000	12.326 30	-4.6	4.108 77	-1.6	25.424 81	-9.5	8.474 94	-3.2
10	5 997 000	12.325 84	-4.6	4.108 61	-1.5	25.423 86	-9.4	8.474 62	-3.1
20	5 978 000	12.325 38	-4.6	4.108 46	-1.5	25.422 92	-9.5	8.474 31	-3.2
30	5 960 000	12.324 92	-4.6	4.108 31	-1.6	25.421 97	-9.5	8.473 99	-3.2
40	5 941 000	12.324 46	-4.6	4.108 15	-1.5	25.421 02	-9.5	8.473 67	-3.1
50	5 923 000	12.324 00	-4.7	4.108 00	-1.6	25.420 07	-9.5	8.473 36	-3.2
37 00	5 904 000	12.323 53	-4.6	4.107 84	-1.5	25.419 12	-9.6	8.473 04	-3.2
10	5 886 000	12.323 07	-4.6	4.107 69	-1.5	25.418 16	-9.6	8.472 72	-3.2
20	5 867 000	12.322 61	-4.7	4.107 54	-1.6	25.417 20	-9.5	8.472 40	-3.2
30	5 849 000	12.322 14	-4.6	4.107 38	-1.5	25.416 25	-9.6	8.472 08	-3.2
40	5 830 000	12.321 68	-4.7	4.107 23	-1.6	25.415 29	-9.6	8.471 76	-3.2
50	5 812 000	12.321 21	-4.6	4.107 07	-1.5	25.414 33	-9.7	8.471 44	-3.2
38 00	5 793 000	12.320 75	-4.7	4.106 92	-1.6	25.413 36	-9.6	8.471 12	-3.2
10	5 775 000	12.320 28	-4.7	4.106 76	-1.6	25.412 40	-9.6	8.470 80	-3.2
20	5 756 000	12.319 81	-4.7	4.106 60	-1.5	25.411 44	-9.7	8.470 48	-3.2
30	5 738 000	12.319 34	-4.7	4.106 45	-1.6	25.410 47	-9.7	8.470 16	-3.3
40	5 719 000	12.318 87	-4.7	4.106 29	-1.6	25.409 50	-9.7	8.469 83	-3.2
50	5 701 000	12.318 40	-4.7	4.106 13	-1.5	25.408 53	-9.7	8.469 51	-3.2
39 00	5 682 000	12.317 93	-4.7	4.105 98	-1.6	25.407 56	-9.7	8.469 19	-3.3
10	5 664 000	12.317 46	-4.7	4.105 82	-1.6	25.406 59	-9.7	8.468 86	-3.2
20	5 645 000	12.316 99	-4.7	4.105 66	-1.5	25.405 62	-9.7	8.468 54	-3.2
30	5 627 000	12.316 52	-4.7	4.105 51	-1.6	25.404 65	-9.8	8.468 22	-3.3
40	5 608 000	12.316 05	-4.7	4.105 35	-1.6	25.403 67	-9.7	8.467 89	-3.2
50	5 590 000	12.315 58	-4.8	4.105 19	-1.6	25.402 70	-9.8	8.467 57	-3.3
40 00	5 571 000	12.315 10	-4.7	4.105 03	-1.5	25.401 72	-9.8	8.467 24	-3.3
10	5 553 000	12.314 63	-4.8	4.104 88	-1.6	25.400 74	-9.8	8.466 91	-3.2
20	5 534 000	12.314 15	-4.7	4.104 72	-1.6	25.399 76	-9.7	8.466 59	-3.3
30	5 516 000	12.313 68	-4.8	4.104 56	-1.6	25.398 79	-9.8	8.466 26	-3.2
40	5 497 000	12.313 20	-4.7	4.104 40	-1.6	25.397 81	-9.9	8.465 94	-3.3
50	5 479 000	12.312 73	-4.8	4.104 24	-1.6	25.396 82	-9.8	8.465 61	-3.3
41 00	5 460 000	12.312 25	-4.7	4.104 08	-1.5	25.395 84	-9.8	8.465 28	-3.3
10	5 442 000	12.311 78	-4.8	4.103 93	-1.6	25.394 86	-9.8	8.464 95	-3.2
20	5 423 000	12.311 30	-4.8	4.103 77	-1.6	25.393 88	-9.9	8.464 63	-3.3
30	5 405 000	12.310 82	-4.8	4.103 61	-1.6	25.392 89	-9.8	8.464 30	-3.3
40	5 386 000	12.310 34	-4.7	4.103 45	-1.6	25.391 91	-9.9	8.463 97	-3.3
50	5 368 000	12.309 87	-4.8	4.103 29	-1.6	25.390 92	-9.8	8.463 64	-3.3
42 00	5 349 000	12.309 39	-4.8	4.103 13	-1.6	25.389 94	-9.9	8.463 31	-3.3
10	5 331 000	12.308 91	-4.8	4.102 97	-1.6	25.388 95	-9.9	8.462 98	-3.3
20	5 312 000	12.308 43	-4.8	4.102 81	-1.6	25.387 96	-9.9	8.462 65	-3.3
30	5 294 000	12.307 95	-4.8	4.102 65	-1.6	25.386 97	-9.8	8.462 32	-3.2
40	5 275 000	12.307 47	-4.8	4.102 49	-1.6	25.385 99	-9.9	8.462 00	-3.3
50	5 257 000	12.306 99	-4.8	4.102 33	-1.6	25.385 00	-9.9	8.461 67	-3.3
43 00	5 238 000	12.306 51	-4.8	4.102 17	-1.6	25.384 01	-9.9	8.461 34	-3.3
10	5 220 000	12.306 03	-4.8	4.102 01	-1.6	25.383 02	-9.9	8.461 01	-3.3
20	5 201 000	12.305 55	-4.8	4.101 85	-1.6	25.382 03	-9.9	8.460 68	-3.3
30	5 183 000	12.305 07	-4.8	4.101 69	-1.6	25.381 04	-9.9	8.460 35	-3.3
40	5 164 000	12.304 59	-4.8	4.101 53	-1.6	25.380 05	-9.9	8.460 02	-3.3
50	5 146 000	12.304 11	-4.8	4.101 37	-1.6	25.379 06	-9.9	8.459 69	-3.3
44 00	5 127 000	12.303 63	-4.8	4.101 21	-1.6	25.378 07	-9.9	8.459 36	-3.3
10	5 109 000	12.303 15	-4.8	4.101 05	-1.6	25.377 08	-10.0	8.459 03	-3.4
20	5 090 000	12.302 67	-4.8	4.100 89	-1.6	25.376 08	-9.9	8.458 69	-3.3
30	5 071 000	12.302 19	-4.8	4.100 73	-1.6	25.375 09	-9.9	8.458 36	-3.3
40	5 053 000	12.301 71	-4.8	4.100 57	-1.6	25.374 10	-9.9	8.458 03	-3.3
50	5 034 000	12.301 23	-4.8	4.100 41	-1.6	25.373 11	-9.9	8.457 70	-3.3

NOTE: Northings are approximate values rounded to the nearest 1,000 metres

LATITUDE FUNCTIONS FOR POINT TO POINT COMPUTATION

LATITUDE	NORTHING	$10^{15}/2r^2$	Diff 1′	$10^{15}/6r^2$	Diff 1′	$10^{10}/2r^2\sin 1''$	Diff 1′	$10^{10}/6r^2\sin 1''$	Diff 1′
45 00	5 016 000	12.300 75	-4.8	4.100 25	-1.6	25.372 12	-9.9	8.457 37	-3.3
10	4 997 000	12.300 27	-4.8	4.100 09	-1.6	25.371 13	-9.9	8.457 04	-3.3
20	4 979 000	12.299 79	-4.8	4.099 93	-1.6	25.370 14	-10.0	8.456 71	-3.3
30	4 960 000	12.299 31	-4.8	4.099 77	-1.6	25.369 14	-9.9	8.456 38	-3.3
40	4 942 000	12.298 83	-4.8	4.099 61	-1.6	25.368 15	-9.9	8.456 05	-3.3
50	4 923 000	12.298 35	-4.8	4.099 45	-1.6	25.367 16	-9.9	8.455 72	-3.3
46 00	4 905 000	12.297 87	-4.8	4.099 29	-1.6	25.366 17	-9.9	8.455 39	-3.3
10	4 886 000	12.297 39	-4.8	4.099 13	-1.6	25.365 18	-9.9	8.455 06	-3.3
20	4 868 000	12.296 91	-4.8	4.098 97	-1.6	25.364 19	-9.9	8.454 73	-3.3
30	4 849 000	12.296 43	-4.8	4.098 81	-1.6	25.363 20	-9.9	8.454 40	-3.3
40	4 831 000	12.295 95	-4.8	4.098 65	-1.6	25.362 21	-9.9	8.454 07	-3.3
50	4 812 000	12.295 47	-4.8	4.098 49	-1.6	25.361 22	-9.9	8.453 74	-3.3
47 00	4 794 000	12.294 99	-4.8	4.098 33	-1.6	25.360 23	-9.9	8.453 41	-3.3
10	4 775 000	12.294 51	-4.8	4.098 17	-1.6	25.359 24	-9.9	8.453 08	-3.3
20	4 757 000	12.294 03	-4.8	4.098 01	-1.6	25.358 25	-9.8	8.452 75	-3.3
30	4 738 000	12.293 55	-4.8	4.097 85	-1.6	25.357 27	-9.9	8.452 42	-3.3
40	4 720 000	12.293 07	-4.8	4.097 69	-1.6	25.356 28	-9.9	8.452 09	-3.3
50	4 701 000	12.292 59	-4.8	4.097 53	-1.6	25.355 29	-9.8	8.451 76	-3.2
48 00	4 683 000	12.292 11	-4.7	4.097 37	-1.6	25.354 31	-9.9	8.451 44	-3.3
10	4 664 000	12.291 64	-4.8	4.097 21	-1.6	25.353 32	-9.8	8.451 11	-3.3
20	4 646 000	12.291 16	-4.8	4.097 05	-1.6	25.352 34	-9.9	8.450 78	-3.3
30	4 627 000	12.290 68	-4.7	4.096 89	-1.5	25.351 35	-9.8	8.450 45	-3.3
40	4 608 000	12.290 21	-4.8	4.096 74	-1.6	25.350 37	-9.8	8.450 12	-3.2
50	4 590 000	12.289 73	-4.8	4.096 58	-1.6	25.349 39	-9.9	8.449 80	-3.3
49 00	4 571 000	12.289 25	-4.7	4.096 42	-1.6	25.348 40	-9.8	8.449 47	-3.3
10	4 553 000	12.288 78	-4.8	4.096 26	-1.6	25.347 42	-9.8	8.449 14	-3.3
20	4 534 000	12.288 30	-4.7	4.096 10	-1.6	25.346 44	-9.8	8.448 81	-3.2
30	4 516 000	12.287 83	-4.8	4.095 94	-1.6	25.345 46	-9.7	8.448 49	-3.3
40	4 497 000	12.287 35	-4.7	4.095 78	-1.5	25.344 49	-9.8	8.448 16	-3.2
50	4 479 000	12.286 88	-4.7	4.095 63	-1.6	25.343 51	-9.8	8.447 84	-3.3
50 00	4 460 000	12.286 41	-4.8	4.095 47	-1.6	25.342 53	-9.7	8.447 51	-3.2
10	4 442 000	12.285 93	-4.7	4.095 31	-1.6	25.341 56	-9.8	8.447 19	-3.3
20	4 423 000	12.285 46	-4.7	4.095 15	-1.5	25.340 58	-9.7	8.446 86	-3.2
30	4 405 000	12.284 99	-4.7	4.095 00	-1.6	25.339 61	-9.7	8.446 54	-3.3
40	4 386 000	12.284 52	-4.7	4.094 84	-1.6	25.338 64	-9.7	8.446 21	-3.2
50	4 368 000	12.284 05	-4.7	4.094 68	-1.5	25.337 67	-9.7	8.445 89	-3.2
51 00	4 349 000	12.283 58	-4.7	4.094 53	-1.6	25.336 70	-9.7	8.445 57	-3.3
10	4 331 000	12.283 11	-4.7	4.094 37	-1.6	25.335 73	-9.7	8.445 24	-3.2
20	4 312 000	12.282 64	-4.7	4.094 21	-1.5	25.334 76	-9.6	8.444 92	-3.2
30	4 293 000	12.282 17	-4.7	4.094 06	-1.6	25.333 80	-9.7	8.444 60	-3.2
40	4 275 000	12.281 70	-4.6	4.093 90	-1.5	25.332 83	-9.6	8.444 28	-3.2
50	4 256 000	12.281 24	-4.7	4.093 75	-1.6	25.331 87	-9.6	8.443 96	-3.2
52 00	4 238 000	12.280 77	-4.7	4.093 59	-1.6	25.330 91	-9.7	8.443 64	-3.3
10	4 219 000	12.280 30	-4.6	4.093 43	-1.5	25.329 94	-9.5	8.443 31	-3.1
20	4 201 000	12.279 84	-4.7	4.093 28	-1.6	25.328 99	-9.6	8.443 00	-3.2
30	4 182 000	12.279 37	-4.6	4.093 12	-1.5	25.328 03	-9.6	8.442 68	-3.2
40	4 164 000	12.278 91	-4.6	4.092 97	-1.5	25.327 07	-9.5	8.442 36	-3.2
50	4 145 000	12.278 45	-4.6	4.092 82	-1.6	25.326 12	-9.6	8.442 04	-3.2
53 00	4 127 000	12.277 99	-4.7	4.092 66	-1.5	25.325 16	-9.5	8.441 72	-3.2
10	4 108 000	12.277 52	-4.6	4.092 51	-1.6	25.324 21	-9.5	8.441 40	-3.1
20	4 090 000	12.277 06	-4.6	4.092 35	-1.5	25.323 26	-9.5	8.441 09	-3.2
30	4 071 000	12.276 60	-4.5	4.092 20	-1.5	25.322 31	-9.4	8.440 77	-3.1
40	4 052 000	12.276 15	-4.6	4.092 05	-1.5	25.321 37	-9.5	8.440 46	-3.2
50	4 034 000	12.275 69	-4.6	4.091 90	-1.6	25.320 42	-9.4	8.440 14	-3.1
54 00	4 015 000	12.275 23	-4.6	4.091 74	-1.5	25.319 48	-9.4	8.439 83	-3.2
10	3 997 000	12.274 77	-4.5	4.091 59	-1.5	25.318 54	-9.4	8.439 51	-3.1
20	3 978 000	12.274 32	-4.6	4.091 44	-1.5	25.317 60	-9.4	8.439 20	-3.1
30	3 960 000	12.273 86	-4.5	4.091 29	-1.5	25.316 66	-9.3	8.438 89	-3.1
40	3 941 000	12.273 41	-4.5	4.091 14	-1.5	25.315 73	-9.4	8.438 58	-3.2
50	3 923 000	12.272 96	-4.5	4.090 99	-1.5	25.314 79	-9.3	8.438 26	-3.1
55 00	3 904 000	12.272 51	-4.5	4.090 84	-1.5	25.313 86	-9.3	8.437 95	-3.1
10	3 886 000	12.272 06	-4.5	4.090 69	-1.5	25.312 93	-9.2	8.437 64	-3.0
20	3 867 000	12.271 61	-4.5	4.090 54	-1.5	25.312 01	-9.3	8.437 34	-3.1
30	3 849 000	12.271 16	-4.5	4.090 39	-1.5	25.311 08	-9.2	8.437 03	-3.1
40	3 830 000	12.270 71	-4.5	4.090 24	-1.5	25.310 16	-9.2	8.436 72	-3.1
50	3 811 000	12.270 26	-4.5	4.090 09	-1.5	25.309 24	-9.2	8.436 41	-3.1

NOTE: Northings are approximate values rounded to the nearest 1,000 metres

POINT SCALE FACTOR

ANNEX G

(AUSTRALIAN MAP GRID)

$k \;=\; k_0\ [1 + (E')^2/2r^2 + (E')^4/24r^4]$

$k \;=\; k_0\ [1 + (XVIII)\ q^2 + 0{\cdot}25\ q^4\ 10^{-4}]$

$E' =$ Easting measured from central meridian in metres

$q \;=\; E'\ 10^{-6}$

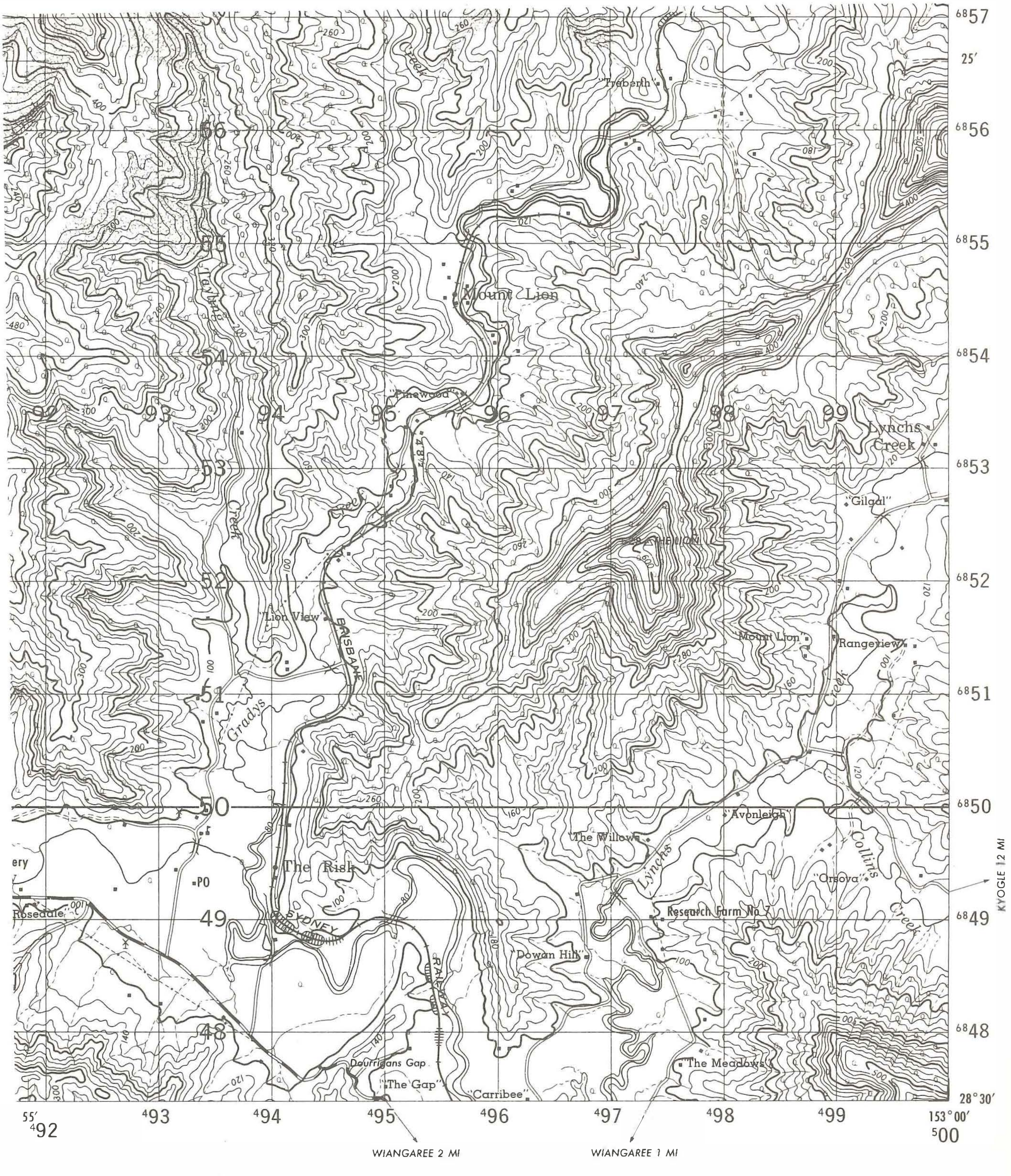
Values of function (XVIII) are tabulated in the US Army Tables

E′	k	E′	k	E′	k	E′	k	E′	k
2 000	0·999 600	62 000	0·999 647	122 000	0·999 783	182 000	1·000 008	242 000	1·000 322
4 000	0·999 600	64 000	0·999 650	124 000	0·999 790	184 000	1·000 017	244 000	1·000 334
6 000	0·999 600	66 000	0·999 654	126 000	0·999 796	186 000	1·000 027	246 000	1·000 346
8 000	0·999 601	68 000	0·999 657	128 000	0·999 802	188 000	1·000 036	248 000	1·000 358
10 000	0·999 601	70 000	0·999 660	130 000	0·999 808	190 000	1·000 045	250 000	1·000 371
12 000	0·999 602	72 000	0·999 664	132 000	0·999 815	192 000	1·000 054	252 000	1·000 383
14 000	0·999 602	74 000	0·999 668	134 000	0·999 821	194 000	1·000 064	254 000	1·000 395
16 000	0·999 603	76 000	0·999 671	136 000	0·999 828	196 000	1·000 074	256 000	1·000 408
18 000	0·999 604	78 000	0·999 675	138 000	0·999 835	198 000	1·000 083	258 000	1·000 421
20 000	0·999 605	80 000	0·999 679	140 000	0·999 842	200 000	1·000 093	260 000	1·000 433
22 000	0·999 606	82 000	0·999 683	142 000	0·999 849	202 000	1·000 103	262 000	1·000 446
24 000	0·999 607	84 000	0·999 687	144 000	0·999 856	204 000	1·000 113	264 000	1·000 459
26 000	0·999 608	86 000	0·999 691	146 000	0·999 863	206 000	1·000 123	266 000	1·000 473
28 000	0·999 610	88 000	0·999 695	148 000	0·999 870	208 000	1·000 133	268 000	1·000 485
30 000	0·999 611	90 000	0·999 700	150 000	0·999 877	210 000	1·000 144	270 000	1·000 499
32 000	0·999 613	92 000	0·999 704	152 000	0·999 885	212 000	1·000 154	272 000	1·000 512
34 000	0·999 614	94 000	0·999 709	154 000	0·999 892	214 000	1·000 165	274 000	1·000 526
36 000	0·999 616	96 000	0·999 714	156 000	0·999 900	216 000	1·000 175	276 000	1·000 539
38 000	0·999 618	98 000	0·999 718	158 000	0·999 908	218 000	1·000 186	278 000	1·000 553
40 000	0·999 620	100 000	0·999 723	160 000	0·999 916	220 000	1·000 197	280 000	1·000 567
42 000	0·999 622	102 000	0·999 728	162 000	0·999 924	222 000	1·000 208	282 000	1·000 580
44 000	0·999 624	104 000	0·999 733	164 000	0·999 932	224 000	1·000 218	284 000	1·000 595
46 000	0·999 626	106 000	0·999 739	166 000	0·999 940	226 000	1·000 230	286 000	1·000 608
48 000	0·999 628	108 000	0·999 744	168 000	0·999 948	228 000	1·000 241	288 000	1·000 623
50 000	0·999 631	110 000	0·999 749	170 000	0·999 956	230 000	1·000 252	290 000	1·000 637
52 000	0·999 633	112 000	0·999 755	172 000	0·999 965	232 000	1·000 264	292 000	1·000 651
54 000	0·999 636	114 000	0·999 760	174 000	0·999 973	234 000	1·000 275	294 000	1·000 666
56 000	0·999 639	116 000	0·999 766	176 000	0·999 982	236 000	1·000 287	296 000	1·000 680
58 000	0·999 641	118 000	0·999 772	178 000	0·999 991	238 000	1·000 298	298 000	1·000 695
60 000	0·999 644	120 000	0·999 778	180 000	0·999 999	240 000	1·000 310	300 000	1·000 710

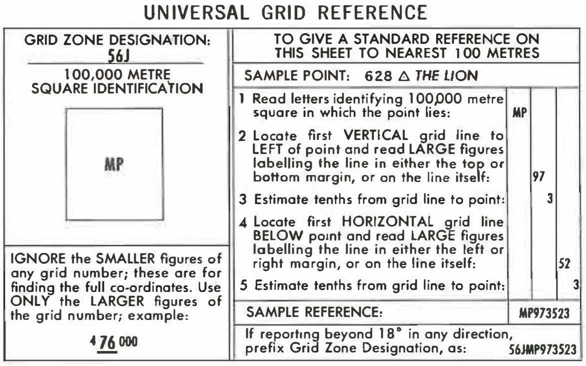
NOTE: This table has been computed for latitude 34° south but may be used through Australia.

Corrections to the table for extreme latitudes are shown below.

E′	150 000	200 000	300 000
LAT 10°S	$+1.10^{-6}$	$+\ 2.10^{-6}$	$+\ 4.10^{-6}$
LAT 44°S	0	$-\ 1.10^{-6}$	$-\ 3.10^{-6}$



ANNEX H: Grid lines covering a map



UNIVERSAL GRID REFERENCE

GRID ZONE DESIGNATION:
56J

100,000 METRE
SQUARE IDENTIFICATION

IGNORE the SMALLER figures of any grid number; these are for finding the full co-ordinates. Use ONLY the LARGER figures of the grid number; example:

4 76 000

TO GIVE A STANDARD REFERENCE ON THIS SHEET TO NEAREST 100 METRES

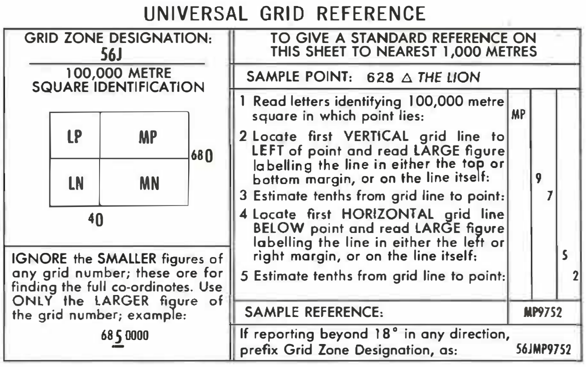
SAMPLE POINT: 628 △ *THE LION*

1 Read letters identifying 100,000 metre square in which the point lies: MP
2 Locate first VERTICAL grid line to LEFT of point and read LARGE figures labelling the line in either the top or bottom margin, or on the line itself: 97
3 Estimate tenths from grid line to point: 3
4 Locate first HORIZONTAL grid line BELOW point and read LARGE figures labelling the line in either the left or right margin, or on the line itself: 52
5 Estimate tenths from grid line to point: 3

SAMPLE REFERENCE: MP973523

If reporting beyond 18° in any direction, prefix Grid Zone Designation, as: 56JMP973523

Six figure references, on maps at 1:100,000 and larger



UNIVERSAL GRID REFERENCE

GRID ZONE DESIGNATION:
56J

100,000 METRE
SQUARE IDENTIFICATION

IGNORE the SMALLER figures of any grid number; these ore for finding the full co-ordinotes. Use ONLY the LARGER figure of the grid number; example:

68 5 0000

TO GIVE A STANDARD REFERENCE ON THIS SHEET TO NEAREST 1,000 METRES

SAMPLE POINT: 628 △ *THE LION*

1 Read letters identifying 100,000 metre square in which point lies: MP
2 Locate first VERTICAL grid line to LEFT of point and read LARGE figure labelling the line in either the top or bottom margin, or on the line itself: 9
3 Estimate tenths from grid line to point: 7
4 Locate first HORIZONTAL grid line BELOW point and read LARGE figure labelling the line in either the left or right margin, or on the line itself: 5
5 Estimate tenths from grid line to point: 2

SAMPLE REFERENCE: MP9752

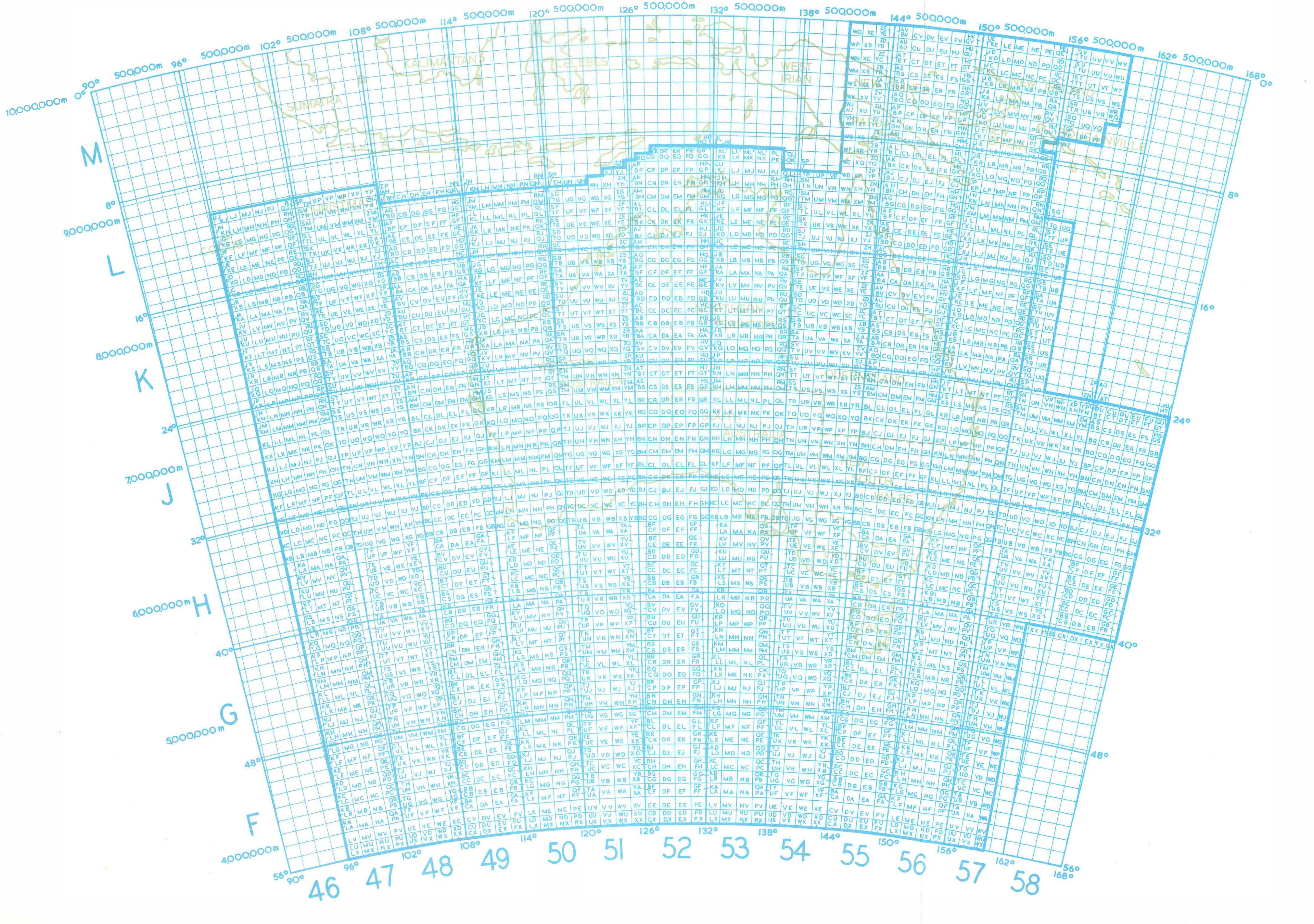
If reporting beyond 18° in any direction, prefix Grid Zone Designation, as: 56JMP9752

Four figure references on a map at1:250,000

Annex I: Grid Reference Boxes

ANNEX J

100,000 METRE SQUARE IDENTIFICATION OF THE AUSTRALIAN MAP GRID AREA



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